

Cold Rydberg atoms

Applications to QIP, many-body physics
and quantum optics

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The “Quantum Optics – Atom” team at Institut d’Optique

<https://atom-tweezers-io.org>



2 projects:

Many-body physics with arrays of individual Rydberg atoms

Light scattering in dense, small cold atomic clouds

A few historical landmarks

1814 Joseph von Fraunhofer

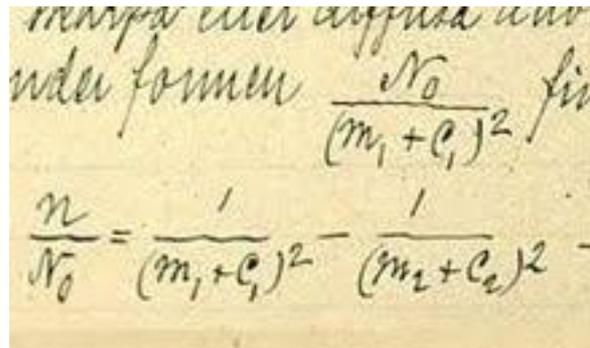


observation of dark lines in spectrum of the sun

1888 “Rydberg formula”



Johannes Rydberg
1854-1919

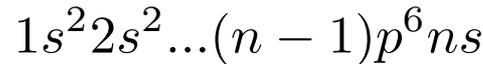
A photograph of a handwritten note on aged paper. The text is in cursive and includes the Rydberg formula:
$$\frac{n}{N_0} = \frac{1}{(m_1 + c_1)^2} - \frac{1}{(m_2 + c_2)^2}$$

$$\frac{1}{\lambda_{nm}} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

Idea of an infinite series
⇒ highly excited states

Periodic Table of the Elements

Alkali: 1 external electron

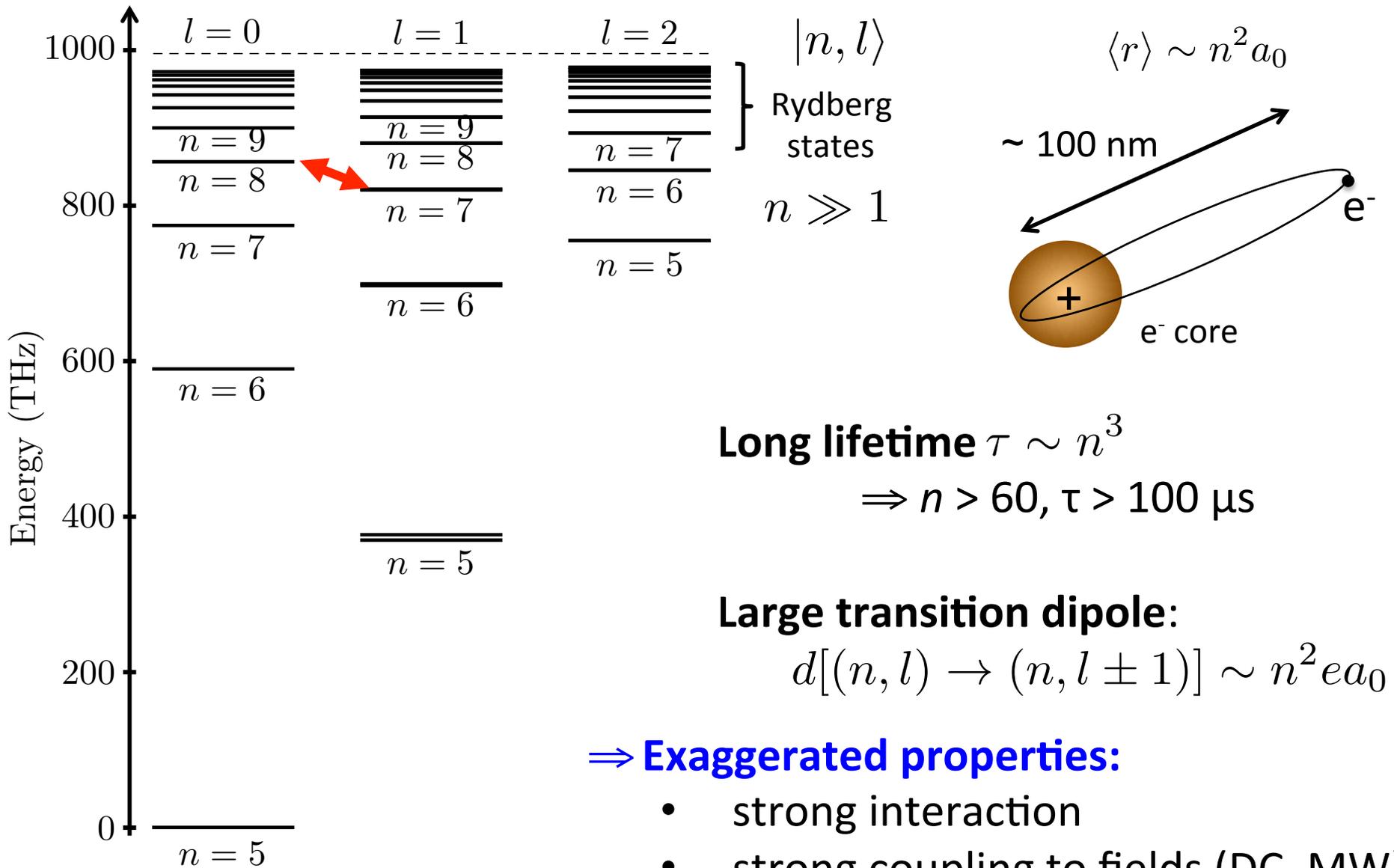


1 H Hydrogen 1.008																	2 He Helium 4.003
3 Li Lithium 6.941	4 Be Beryllium 9.012											5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180
11 Na Sodium 22.990	12 Mg Magnesium 24.305											13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.867	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.845	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.631	33 As Arsenic 74.922	34 Se Selenium 78.971	35 Br Bromine 79.904	36 Kr Krypton 84.798
37 Rb Rubidium 84.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.95	43 Tc Technetium 98.907	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.905	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.414	49 In Indium 114.818	50 Sn Tin 118.711	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.904	54 Xe Xenon 131.294
55 Cs Cesium 132.905	56 Ba Barium 137.328	57-71 Lanthanides	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.217	78 Pt Platinum 195.085	79 Au Gold 196.967	80 Hg Mercury 200.592	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium [208.982]	85 At Astatine 209.987	86 Rn Radon 222.018
87 Fr Francium 223.020	88 Ra Radium 226.025	89-103 Actinides	104 Rf Rutherfordium [261]	105 Db Dubnium [262]	106 Sg Seaborgium [266]	107 Bh Bohrium [264]	108 Hs Hassium [269]	109 Mt Meitnerium [268]	110 Ds Darmstadtium [269]	111 Rg Roentgenium [272]	112 Cn Copernicium [277]	113 Uut Ununtrium unknown	114 Fl Flerovium [289]	115 Uup Ununpentium unknown	116 Lv Livermorium [293]	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown

57 La Lanthanum 138.905	58 Ce Cerium 140.116	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.243	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.500	67 Ho Holmium 164.930	68 Er Erbium 167.259	69 Tm Thulium 168.934	70 Yb Ytterbium 173.055	71 Lu Lutetium 174.967
89 Ac Actinium 227.028	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.048	94 Pu Plutonium 244.064	95 Am Americium 243.061	96 Cm Curium 247.070	97 Bk Berkelium 247.070	98 Cf Californium 251.080	99 Es Einsteinium [254]	100 Fm Fermium 257.095	101 Md Mendelevium 258.1	102 No Nobelium 259.101	103 Lr Lawrencium [262]

- Alkali Metal
- Alkaline Earth
- Transition Metal
- Basic Metal
- Semimetal
- Nonmetal
- Halogen
- Noble Gas
- Lanthanide
- Actinide

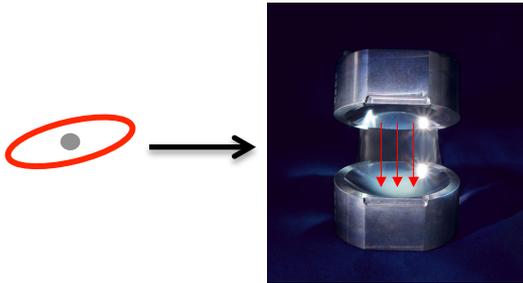
“Rydberg atom” = a highly excited atom (e.g. Rb)



Back to history...

1975 Spectroscopy using lasers (Gallagher, Kleppner, Haroche...)

1980 – 2000 Cavity Quantum Electrodynamics using Rydbergs

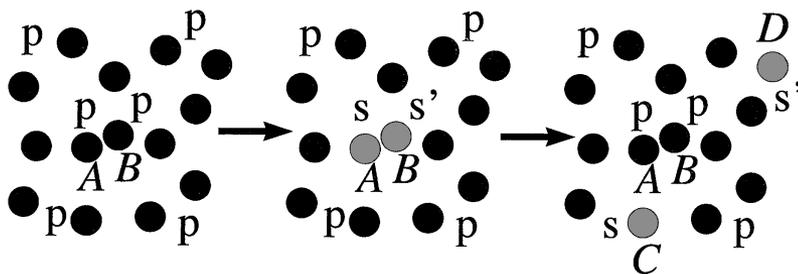


High Q cavity: photon lifetime $> 1\text{ms}$
+ large dipole \Rightarrow
1 Rydberg interacts with 1 photon!

Haroche, Walther...



1998 Rydbergs meet **cold atoms** P. Pillet and T. Gallagher



"Frozen" gas

Anderson, PRL **80**, 249 (1998)

Mourachko, PRL **80**, 253 (1998)

Diffusion of excitation faster
than motion \Rightarrow correlations
between all atoms

$k_B T \ll$ Interaction energy

$\Rightarrow T < 1\text{ mK}$

Interactions between Rydberg atoms

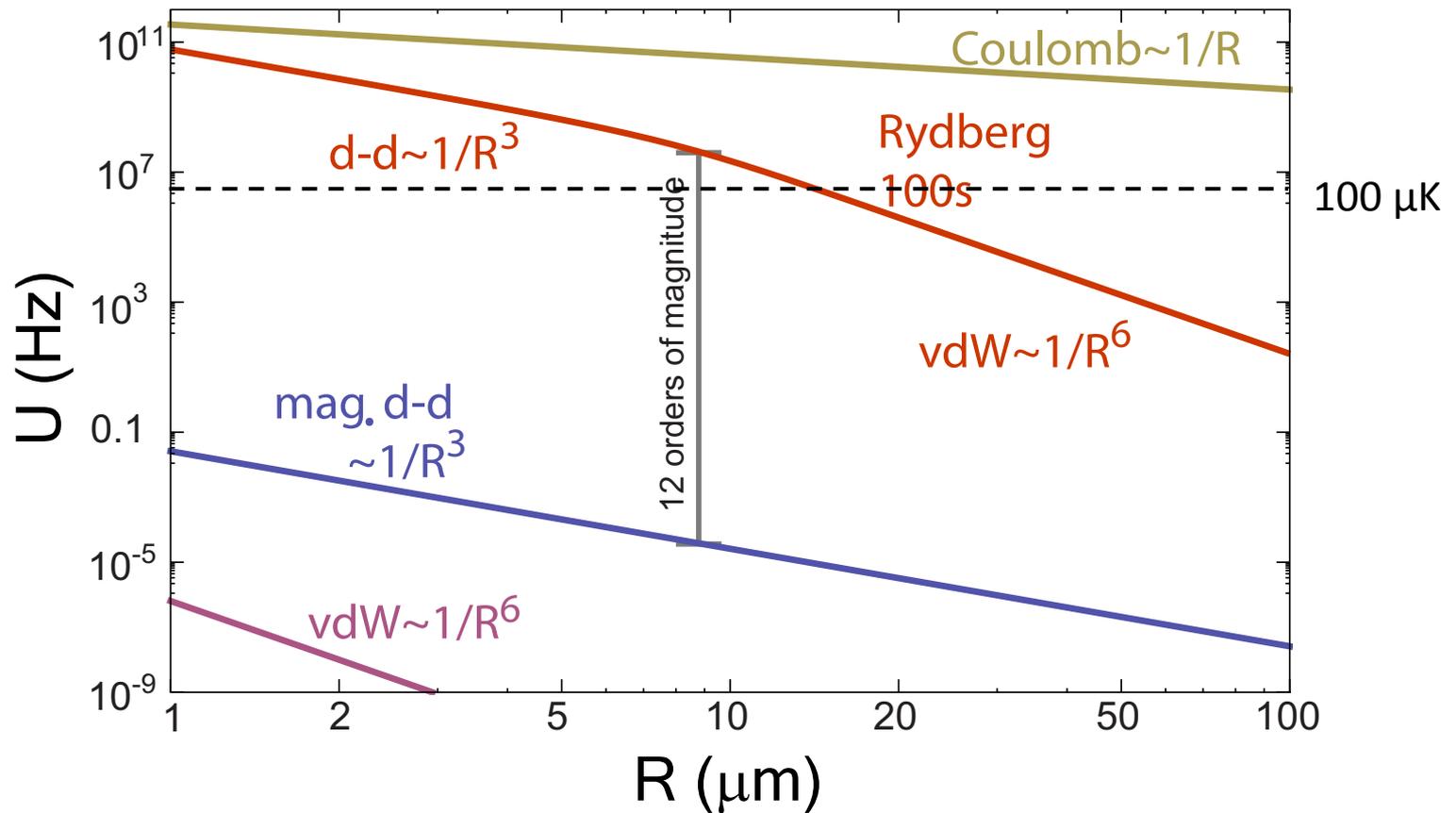
REVIEWS OF MODERN PHYSICS, VOLUME 82, JULY–SEPTEMBER 2010

Quantum information with Rydberg atoms

M. Saffman and T. G. Walker

Department of Physics, University of Wisconsin, 1150 University Avenue, Madison, Wisconsin 53706, USA

K. Mølmer



A new era: the Rydberg Blockade idea

VOLUME 85, NUMBER 10

PHYSICAL REVIEW LETTERS

4 SEPTEMBER 2000

Fast Quantum Gates for Neutral Atoms

D. Jaksch, J. I. Cirac, and P. Zoller

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S. L. Rolston

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VOLUME 87, NUMBER 3

PHYSICAL REVIEW LETTERS

16 JULY 2001

Dipole Blockade and Quantum Information Processing in Mesoscopic Atomic Ensembles

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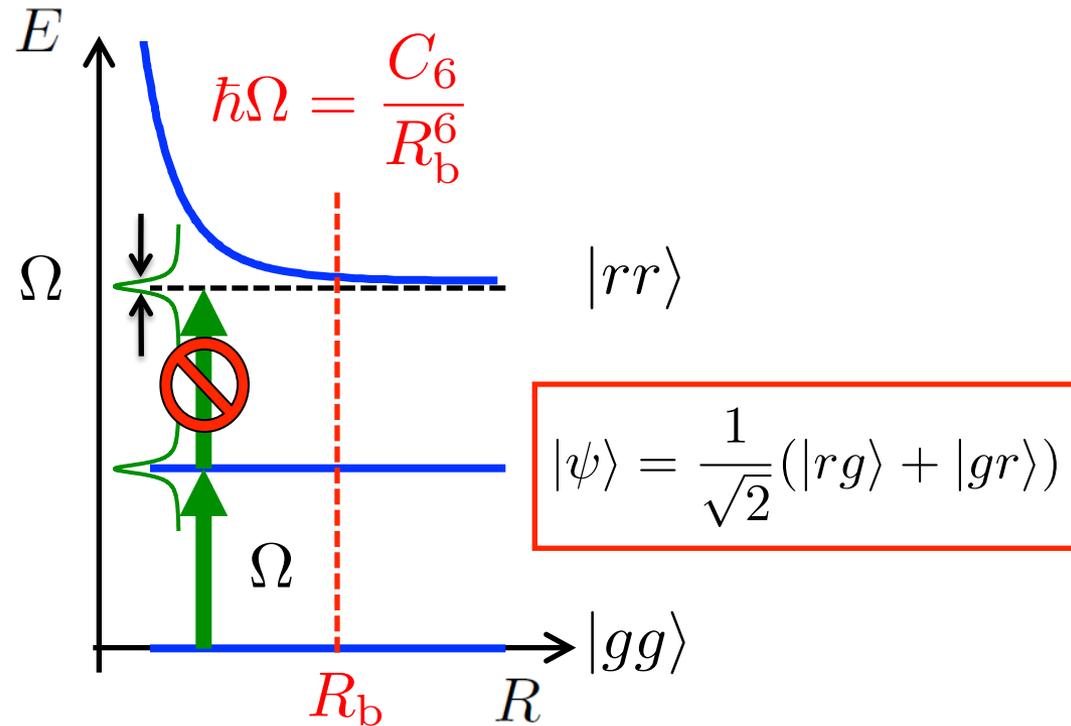
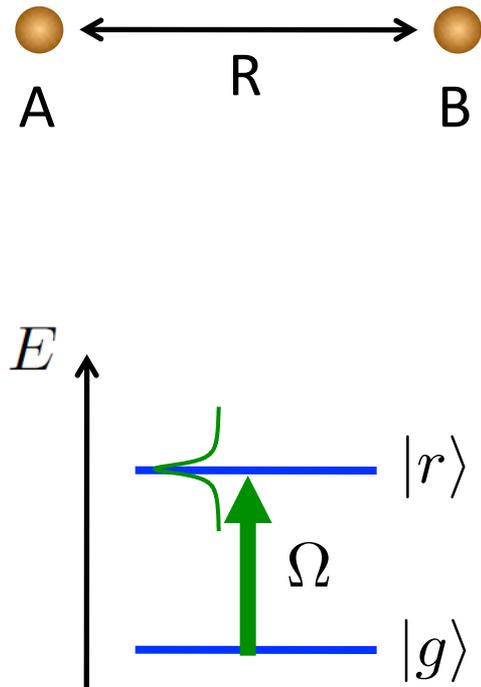
³*Physics Department, University of Connecticut, Storrs, Connecticut 06269*

L. M. Duan, D. Jaksch, J. I. Cirac, and P. Zoller

Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria

(Received 7 November 2000; published 26 June 2001)

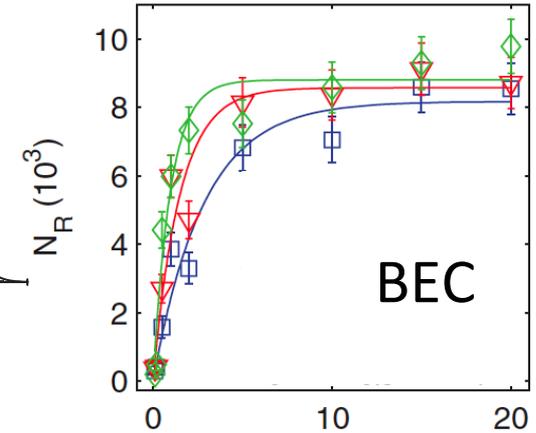
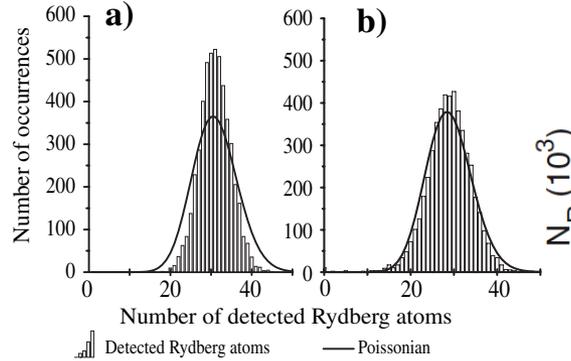
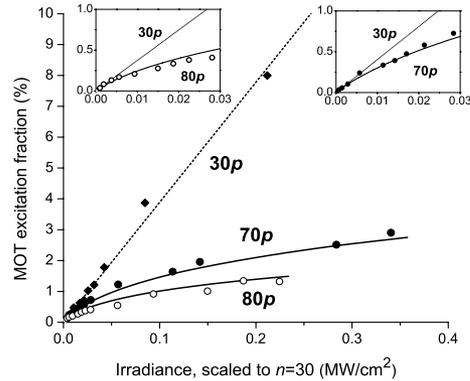
A new era: the Rydberg Blockade idea



Blockade \Rightarrow **entanglement and gates!!**

The first blockade experiments

Atomic ensembles



Gould, PRL 2004

Martin, PRL 2004

Raithel, PRL 2005

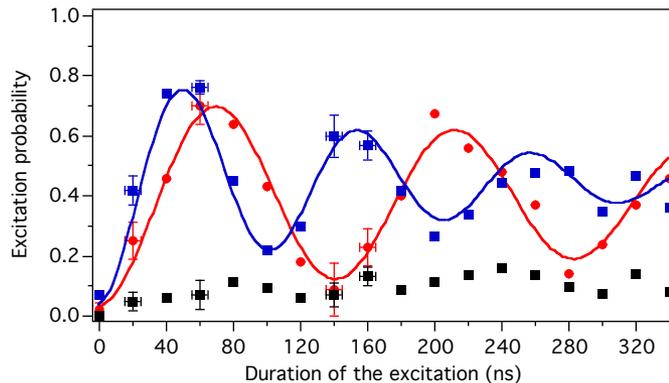
Weidemuller, PRL 2004

Pillet, PRL 2006

Pfau, PRL 2007

Individual atoms

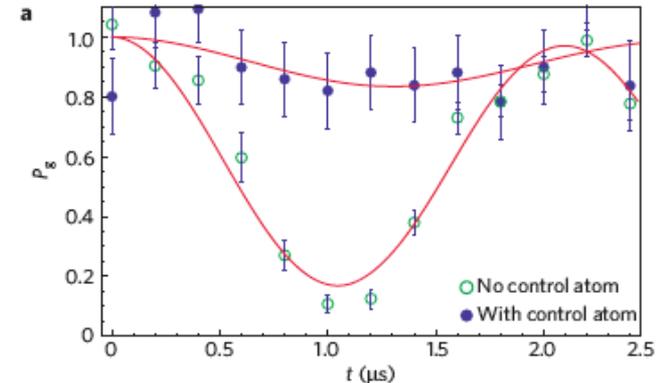
IO Palaiseau



Blockade + collective excitation $\sqrt{2}$

Gaétan *et al.*, Nat. Phys. **5**, 115 (2009)

U. Wisconsin

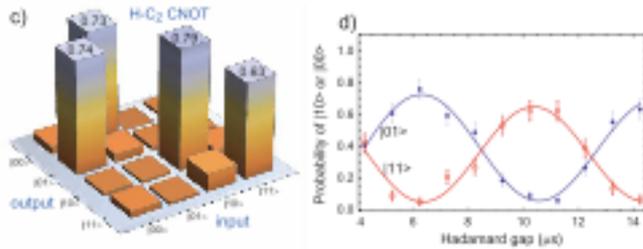


Blockade

Urban *et al.*, Nat. Phys. **5**, 110 (2009)

And now (2017)... a few examples

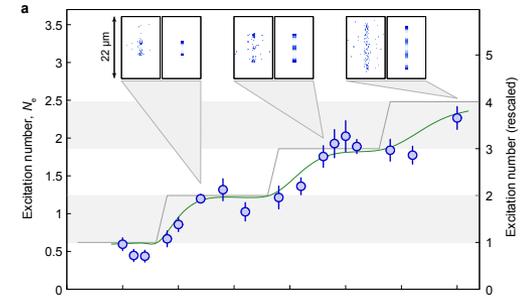
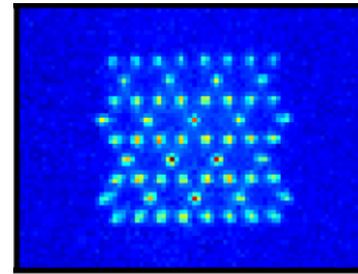
QIP: entanglement and gates



Saffman RMP **82**, 2313 (2010)

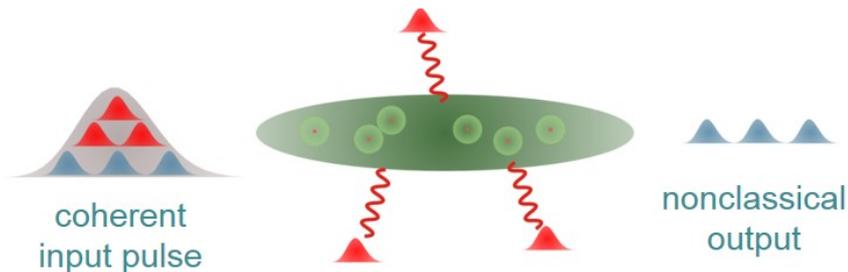
Saffman, Biedermann...

Many-body physics Quantum simulation



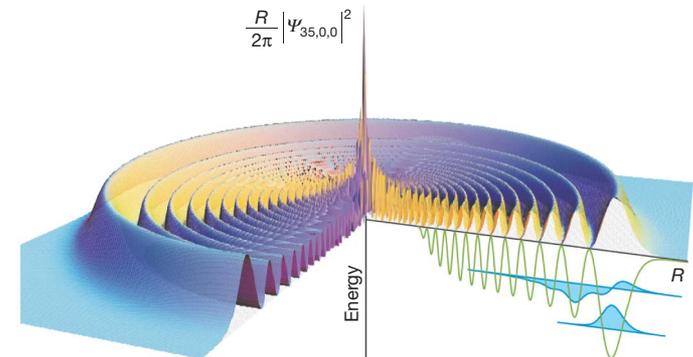
Browaeys, Lukin, Bloch, Pillet, Weidemuller, Morsch...

Non-linear classical & quantum optics



Adams, Hofferbert, Firstenberg, Lukin, Vuletic...

Exotic long-range molecules



Pfau-Löw, Ott, Shaeffer ...

Outline

- Lecture 1: Rydberg atoms and their interactions (~2h)
- Lecture 2: Rydberg blockade and application to QIP (~1h)
- Lecture 3: Quantum simulation & Quantum Optics with Rydbergs (~1h)

Properties of Rydberg atoms

References:

“Rydberg atoms”, T. Gallagher, Cambridge (1994)

“An experimental and theoretical guide to strongly interacting Rydberg gases”, R. Loew, J. Phys. B **45**, 113001(2012)

“Quantum Information with Rydberg atoms”, M. Saffman, T. Walker, K. Moelmer, Rev. Mod. Phys. **82**, 2313 (2010)

Special Issue on Rydberg Atomic Physics, J. Phys. B (2016) contains many reviews

Quantum defects for alkali atoms

Experiments $\Rightarrow E_n = -\frac{R_y}{(n - \delta_{nlj})^2} \quad R_y = R_y^\infty \left(1 + \frac{m_e}{M}\right)^{-1}$

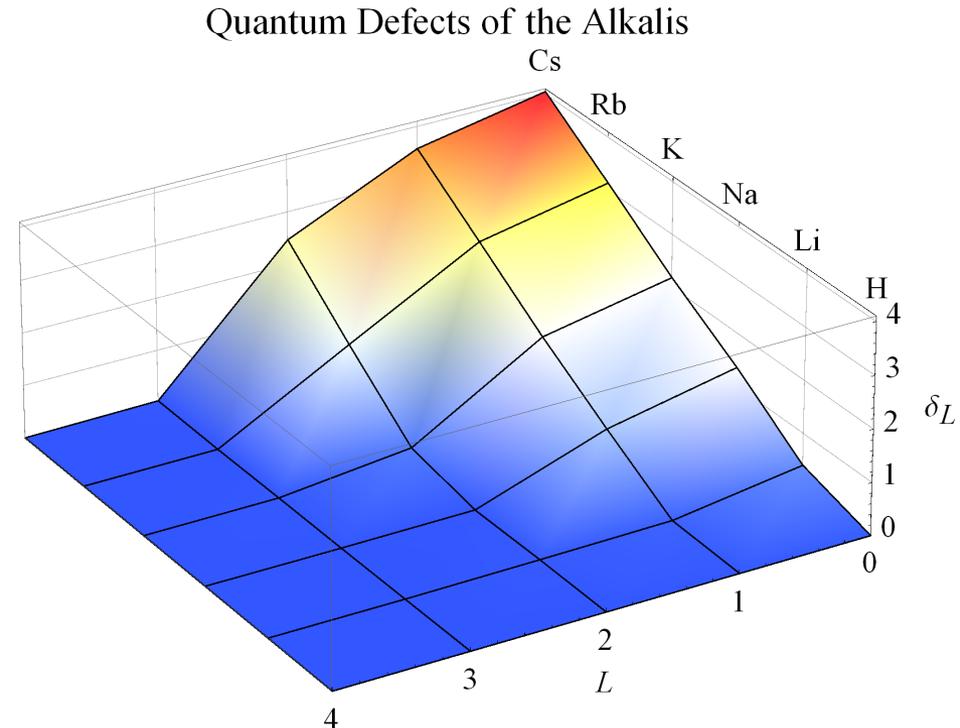
$R_y^\infty = 10\,973\,371.568\,539 \text{ m}^{-1}$

Quantum defects (Experimental)

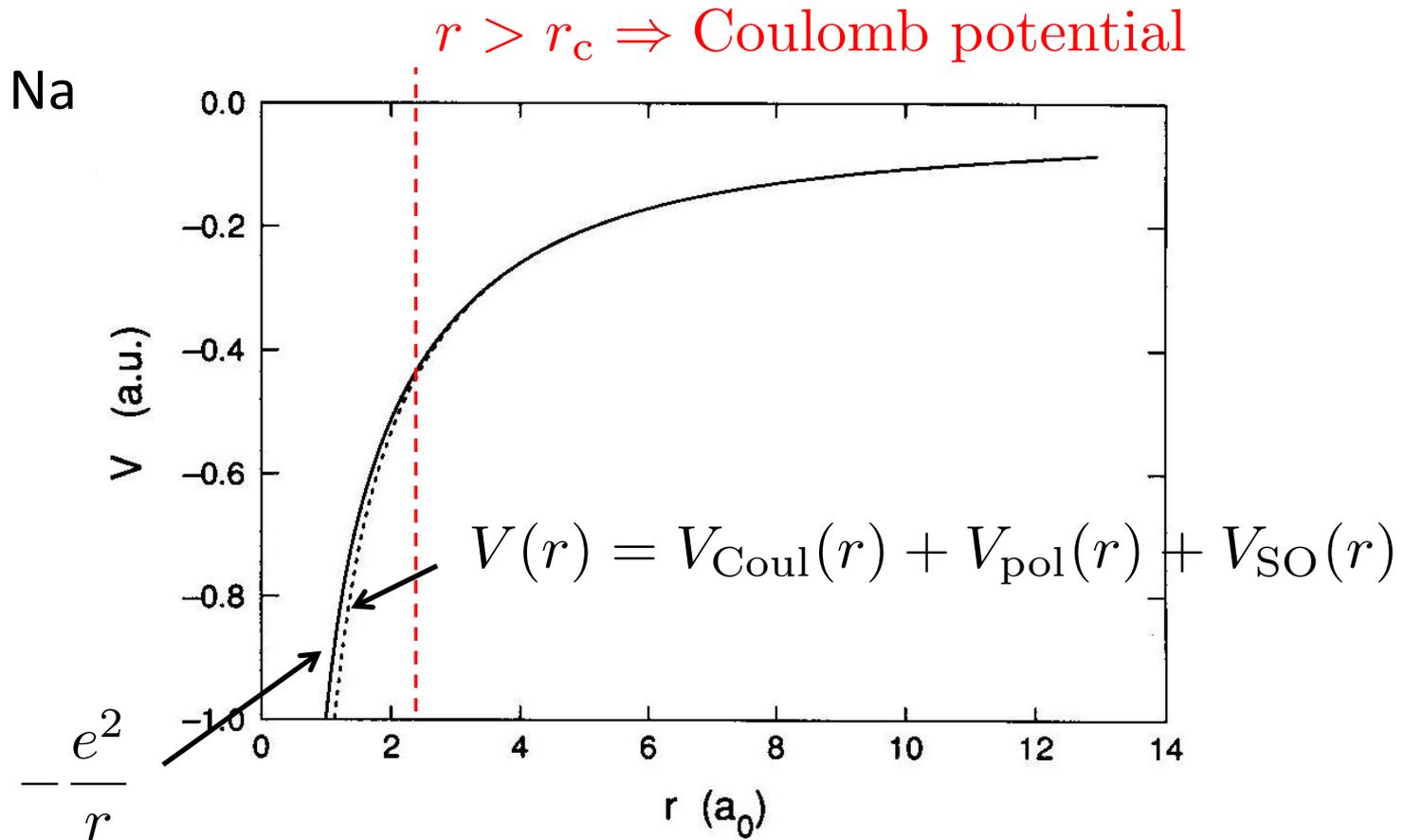
For Rb:

$n \geq 30$

L	J	$\delta_{L,J}$
0	1/2	3.131
1	1/2	2.654
	3/2	2.641
2	3/2	1.348
	5/2	1.346
3	5/2	0.016
	7/2	0.016



The “effective” potential

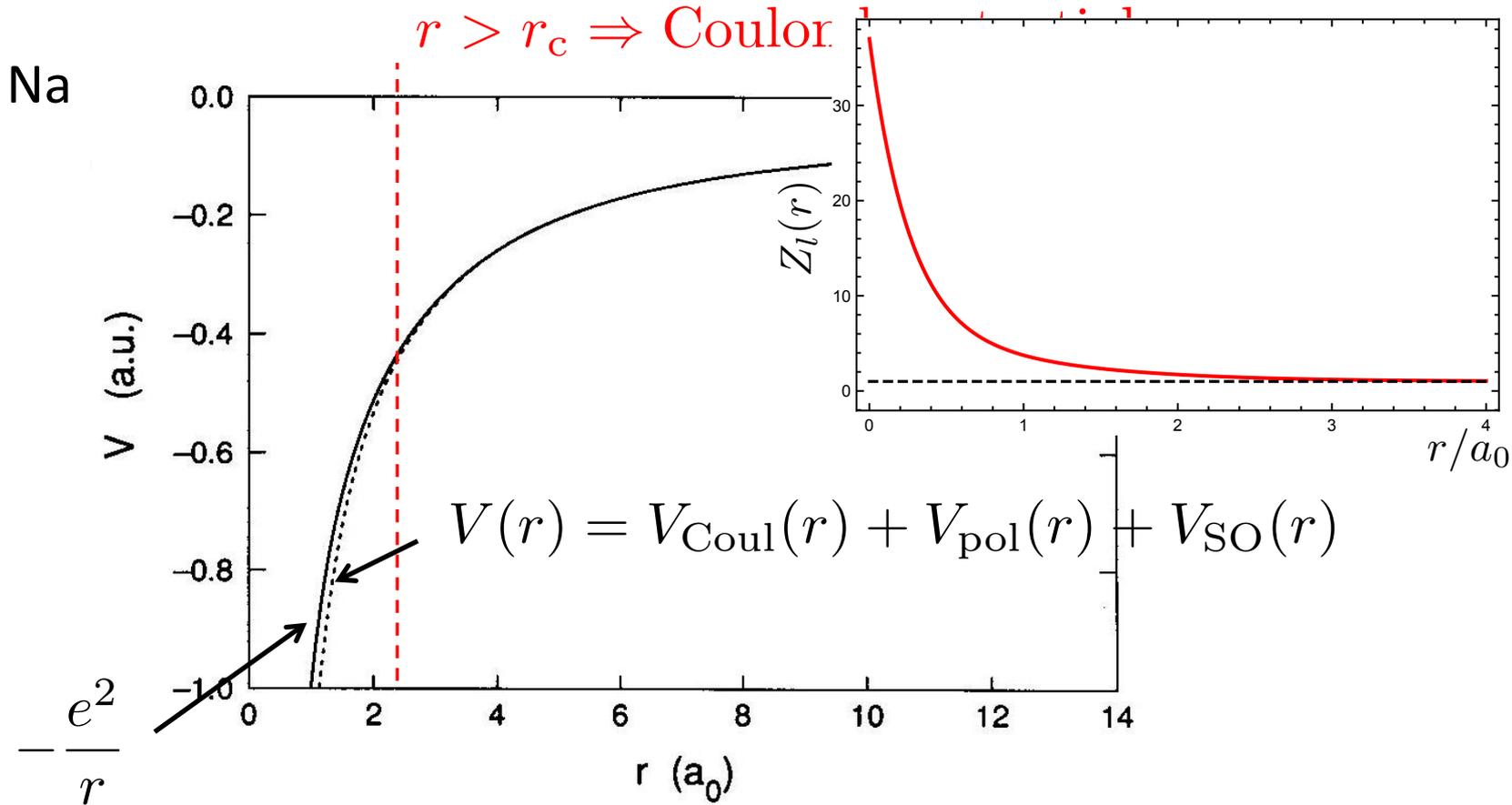


$$V_{\text{Coul}}(r) = -\frac{Z_l(r)e^2}{r}$$

$$V_{\text{pol}}(r) = -\frac{\alpha_D}{2r^4}$$

$$V_{\text{SO}}(r) = \frac{\alpha^2}{2r^3} \mathbf{L} \cdot \mathbf{S}$$

The “effective” potential

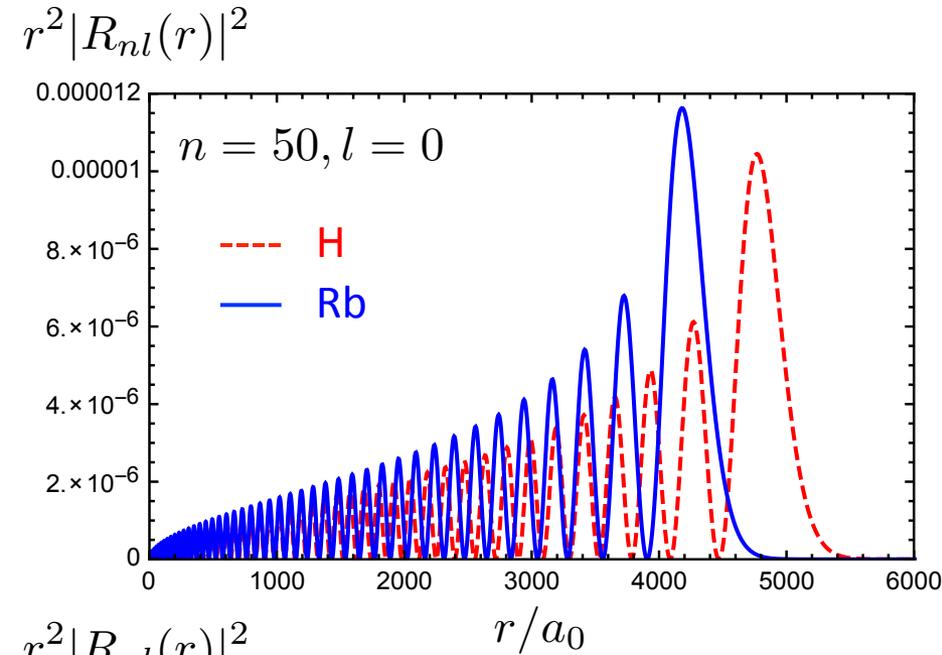


$$V_{\text{Coul}}(r) = -\frac{Z_l(r)e^2}{r}$$

$$V_{\text{pol}}(r) = -\frac{\alpha_D}{2r^4}$$

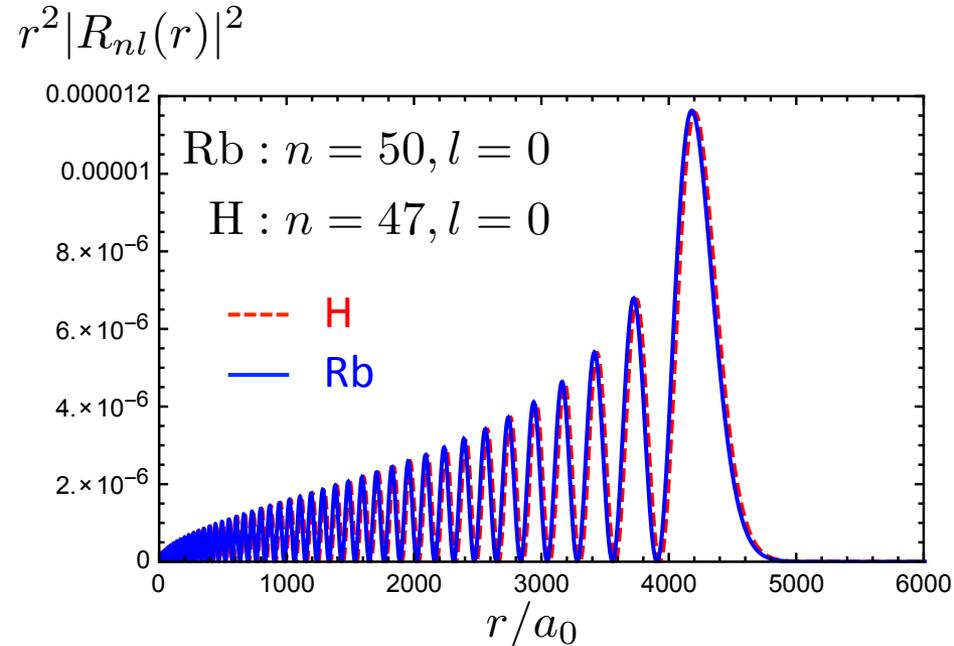
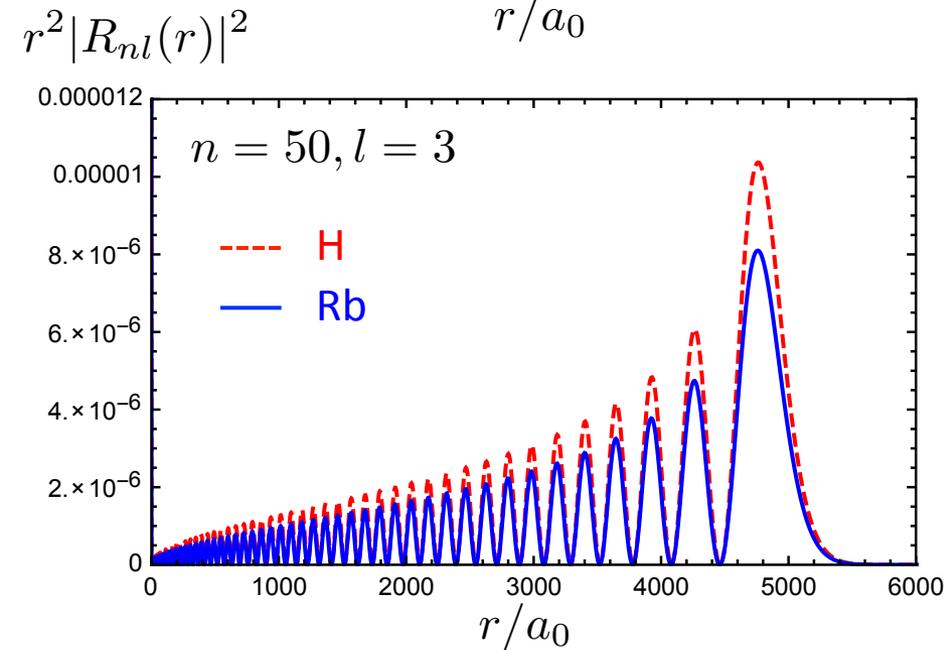
$$V_{\text{SO}}(r) = \frac{\alpha^2}{2r^3} \mathbf{L} \cdot \mathbf{S}$$

Radial wave-function for rubidium



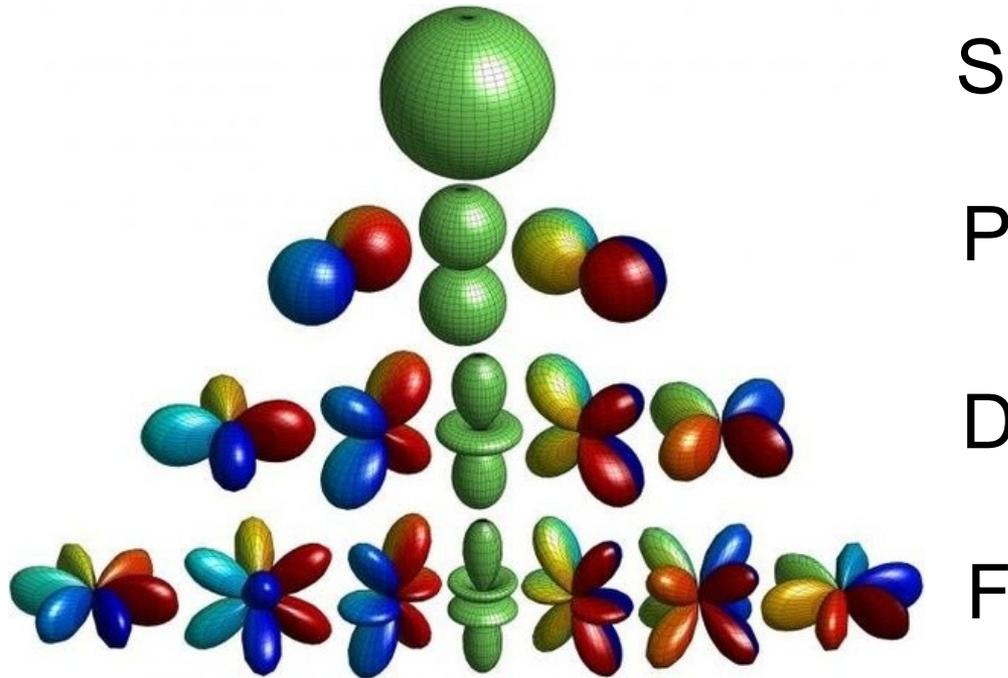
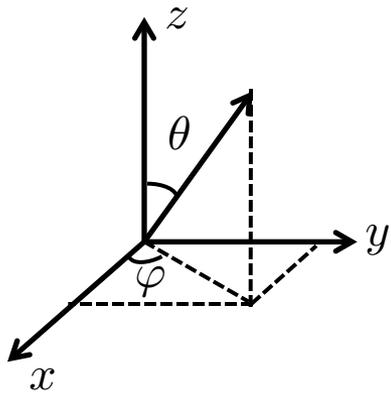
Numerov algorithm
Zimmerman, PRA 20, 2251 (1978)

$$n = 50 \Rightarrow n^* = n - \delta_0$$
$$n^* \approx 50 - 3.13 \approx 47$$

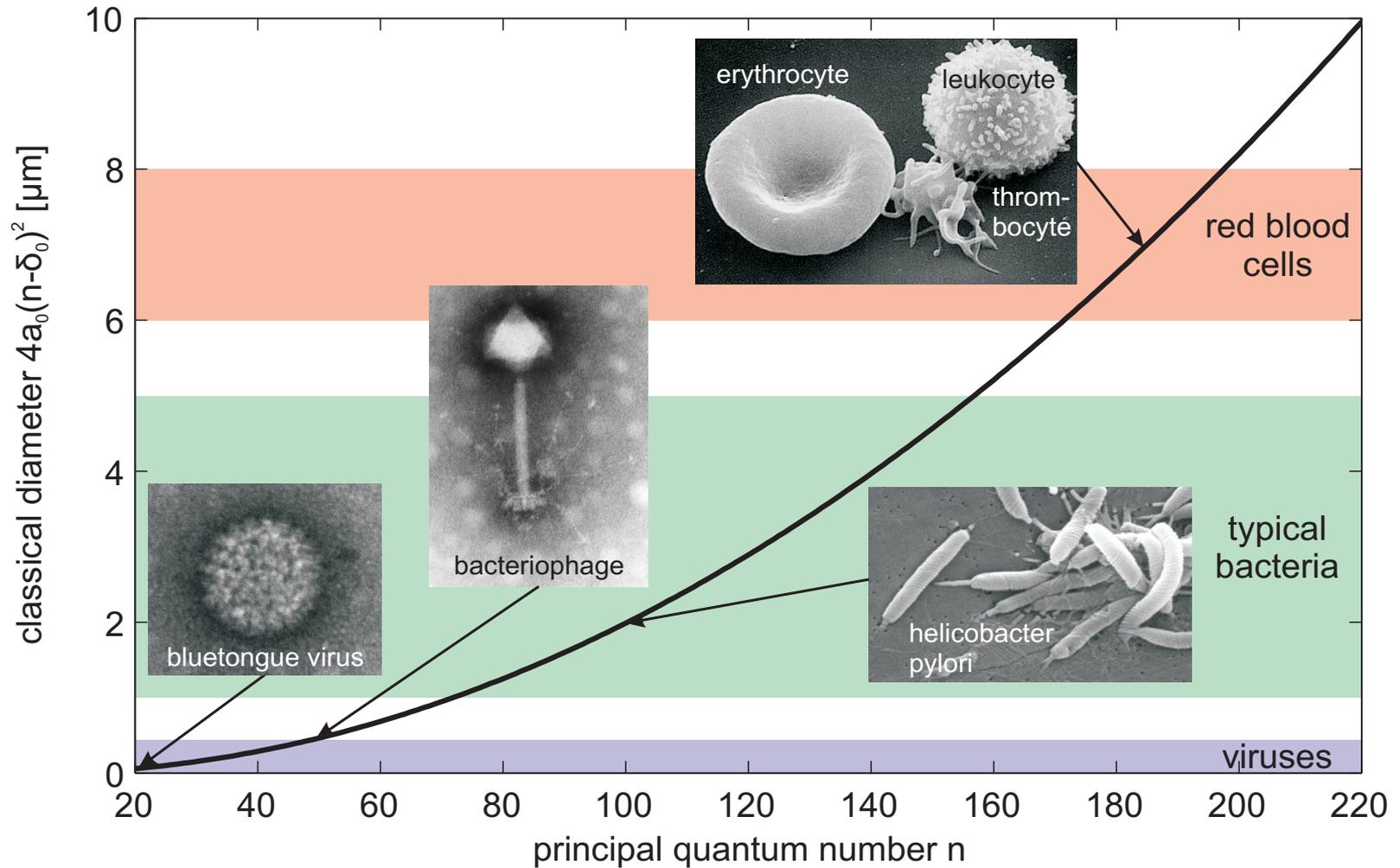


Angular wave-function

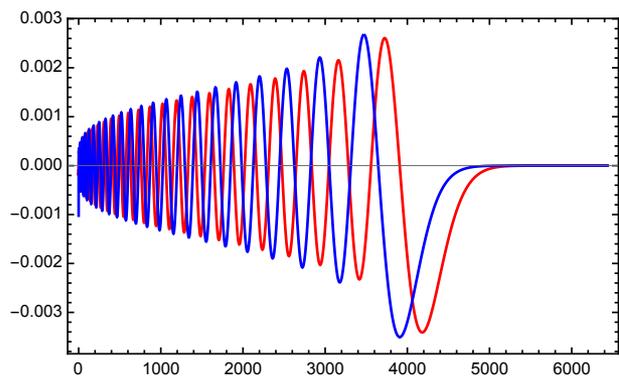
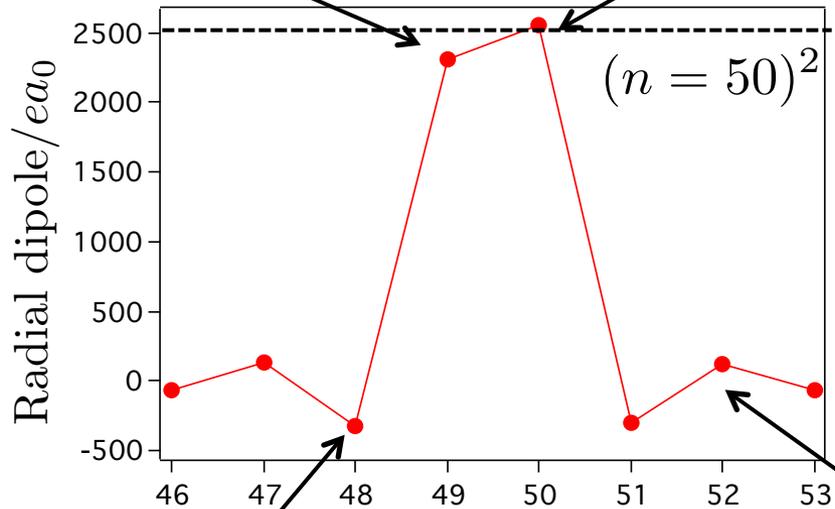
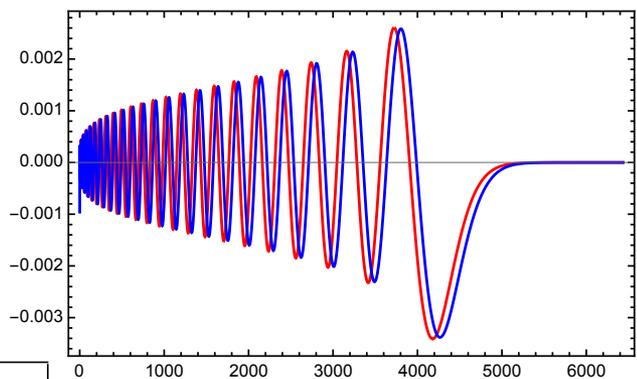
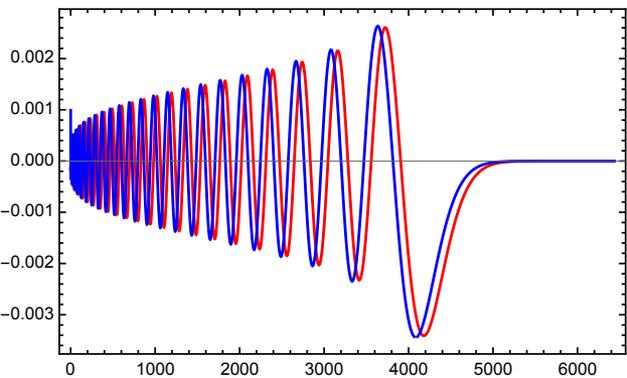
$$|Y_{lm}(\theta, \varphi)|^2$$



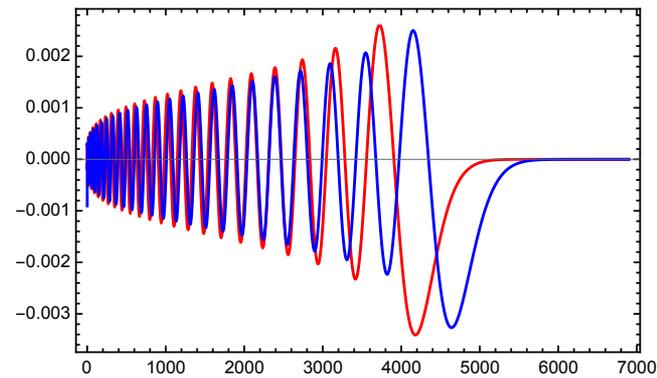
Rydberg atoms are huge...



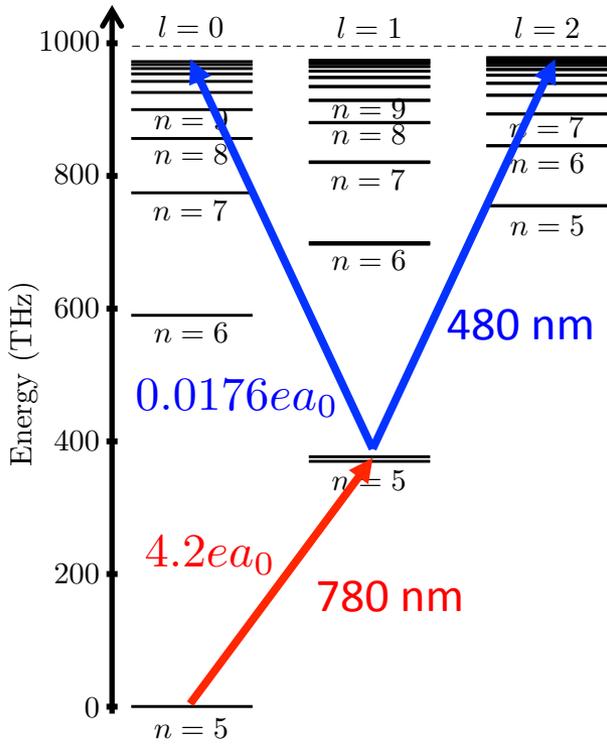
Dipole matrix element from 50s (radial part)



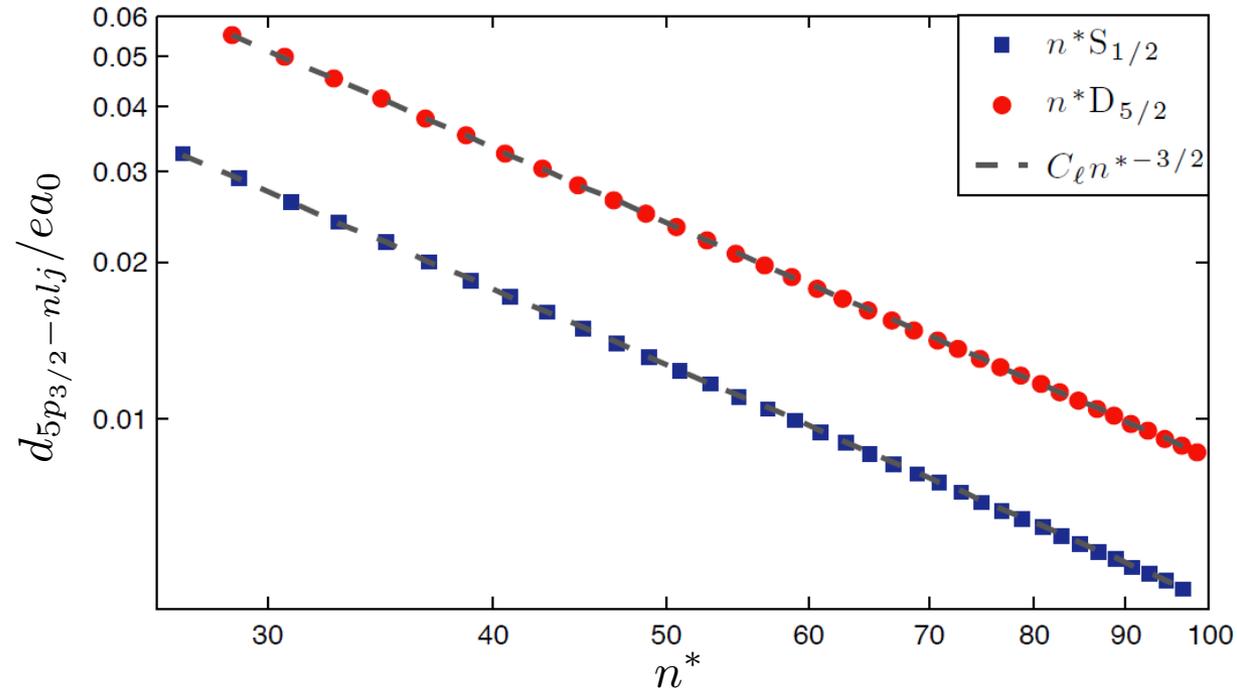
n_{sp}



Dipole matrix element from low lying states

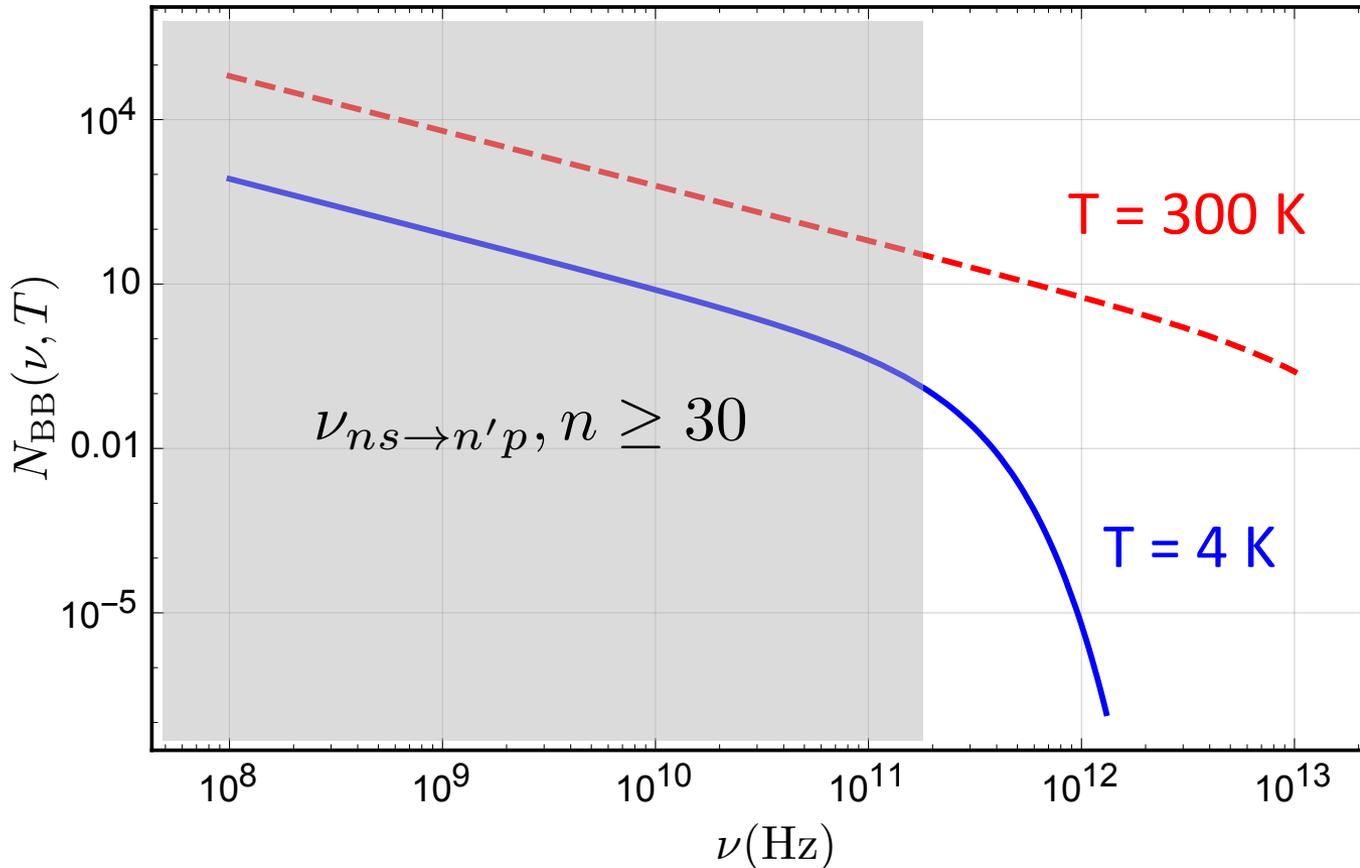


$$d_{5p_{3/2}-nlj} \propto \frac{ea_0}{(n^*)^{\frac{3}{2}}}$$



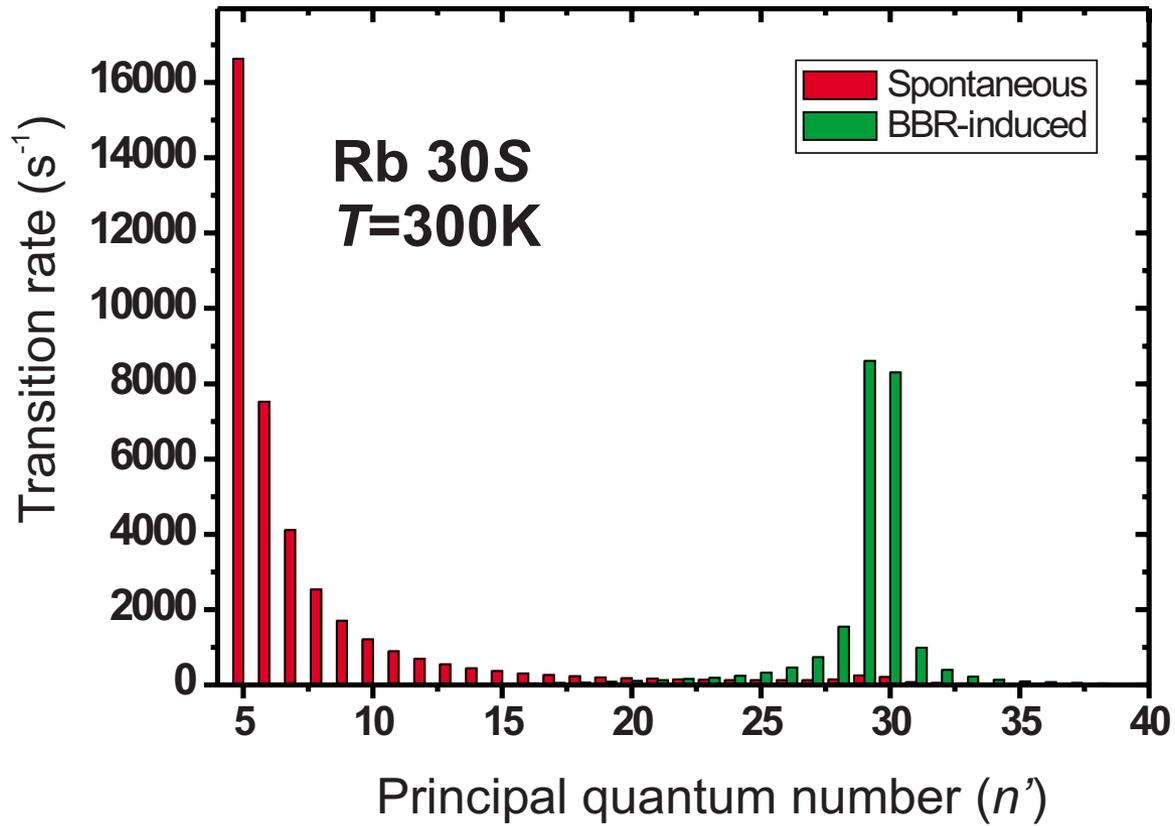
Black-Body radiation

Number of photons / mode:
$$N_{\text{BB}}(\nu, T) = \frac{1}{e^{\frac{h\nu}{k_{\text{B}}T}} - 1}$$

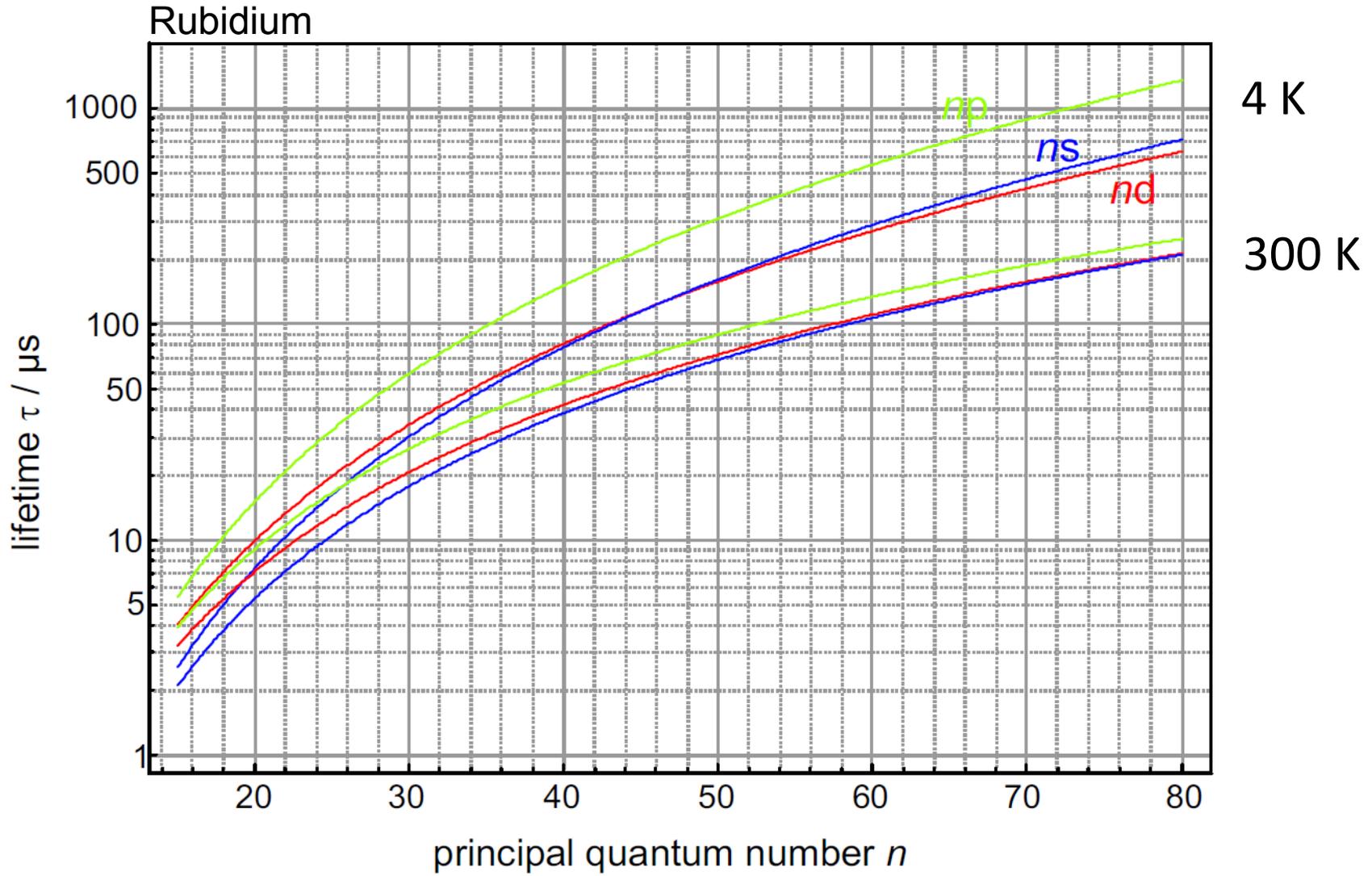


Stimulated rate:
$$\Gamma_{n \rightarrow n'}^{\text{stim, BB}} = \Gamma_{n \rightarrow n'}^{\text{rad}} N_{\text{BB}}(\nu, T)$$

Rydberg lifetimes



Rydberg lifetimes



Stark map without fine structure (Li)

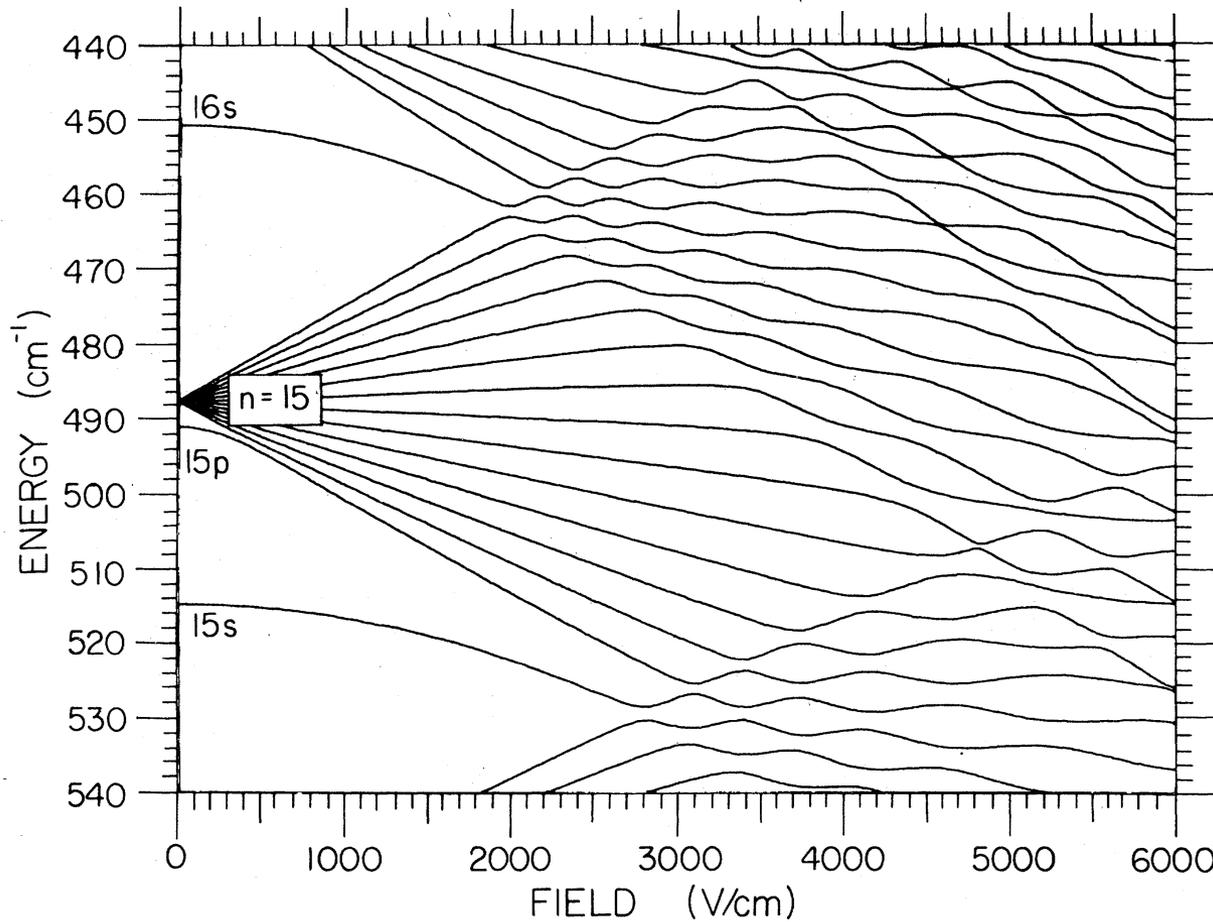


FIG. 6. Lithium, $m=0$.

Summary: Rydberg have exaggerated properties

Table 1. Properties of Rydberg states.

Property	n -scaling	Value for $80S_{1/2}$ of Rb
Binding energy E_n	n^{-2}	−500 GHz
Level spacing $E_{n+1} - E_n$	n^{-3}	13 GHz
Size of wavefunction $\langle r \rangle$	n^2	500 nm
Lifetime τ	n^3	200 μ s
Polarizability α	n^7	−1.8 GHz/(V/cm) ²
van der Waals coefficient C_6	n^{11}	4 THz · μ m ⁶

A few experimental considerations

References:

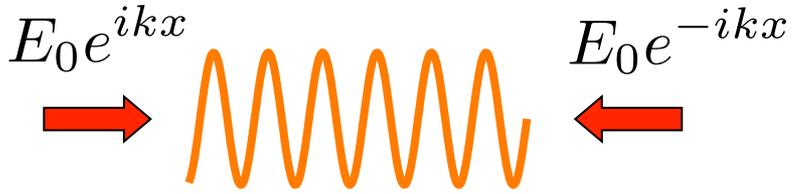
Laser cooling and trapping, Nobel Lectures Phillips, Chu, Cohen-Tannoudji, Rev. Mod. Phys. **70**, july 1998

“An experimental and theoretical guide to strongly interacting Rydberg gases”, R. Loew, J. Phys. B **45**, 113001(2012)

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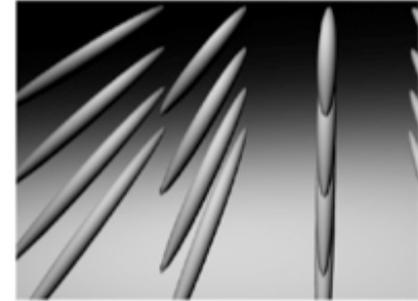
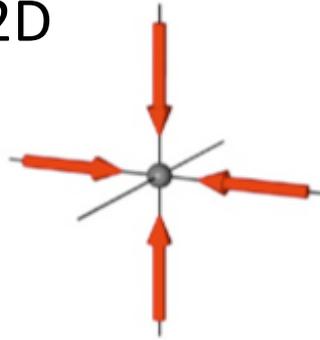
Optical lattices

1D

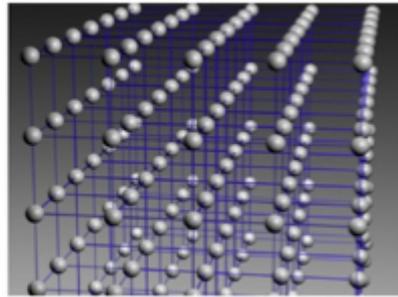
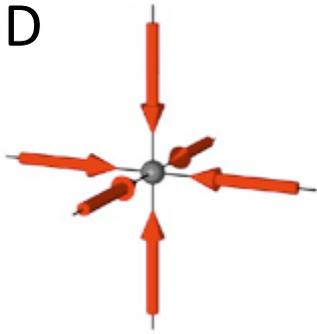


$$I(x) = 2E_0^2(1 + \cos 2kx)$$

2D



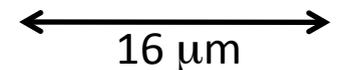
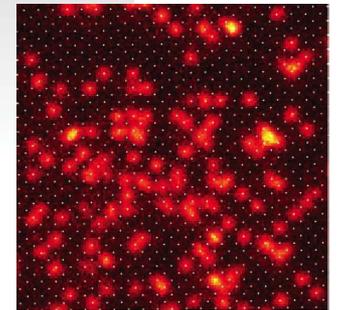
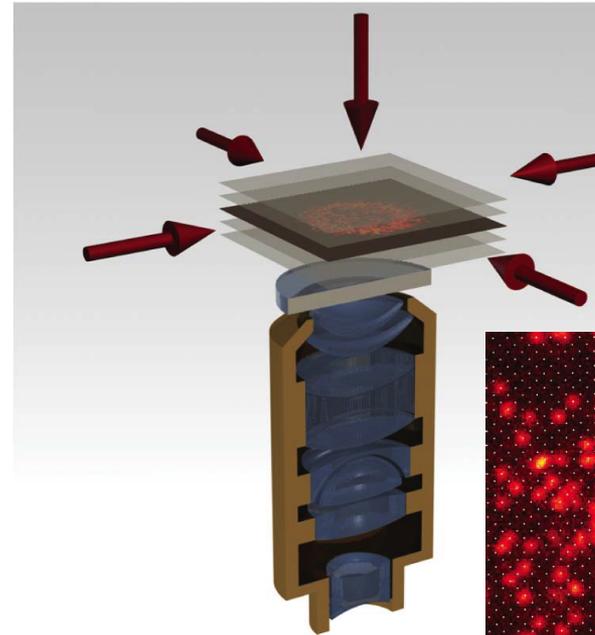
3D



$$\lambda/2 = 0.5 \mu\text{m}$$

(M. Greiner thesis)

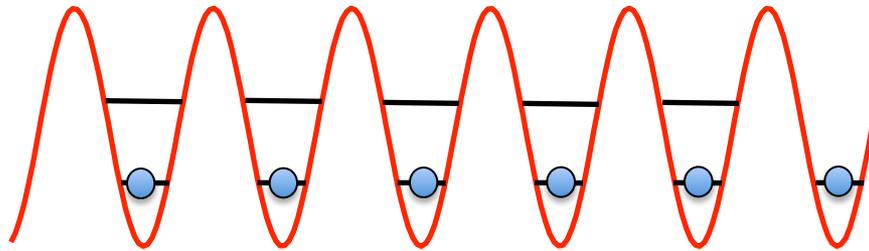
Single site resolution ($< 1 \mu\text{m}$)



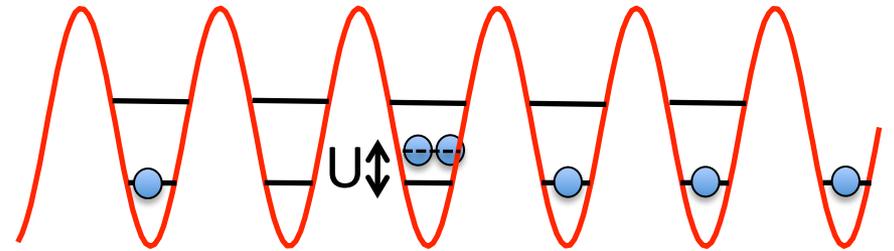
Bakr *et al.*, Nature **462**, 74 (2009)
Sherson *et al.*, Nature **467**, 68 (2010)

Preparation of individual atoms in optical lattices

Superfluid - Mott insulator transition Greiner *et al.*, Nature **415**, 39 (2002)



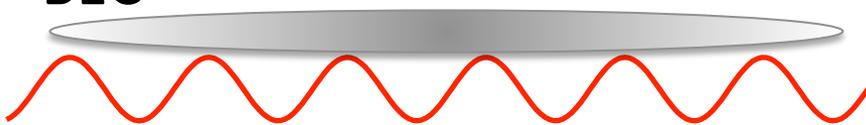
$$6E_0$$



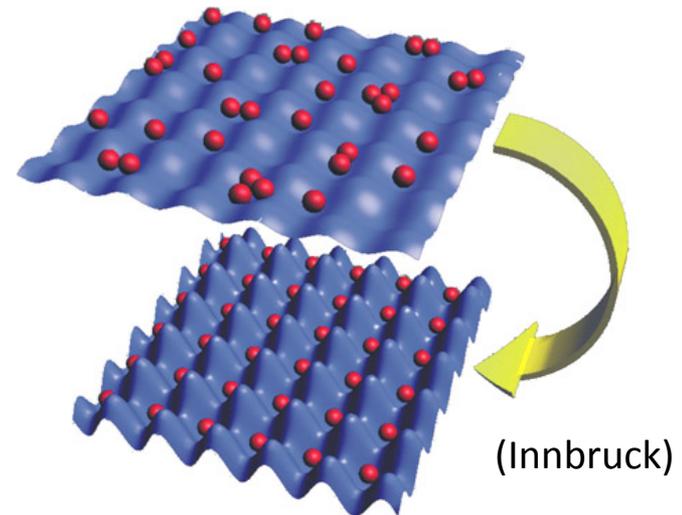
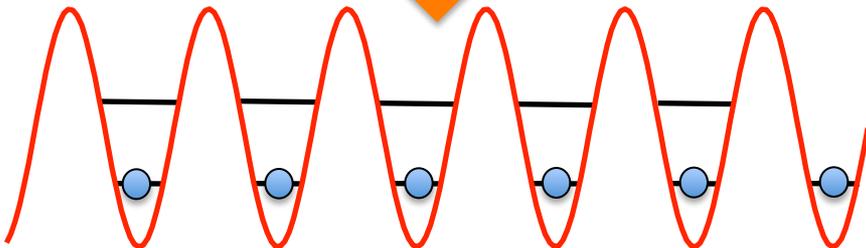
$$6E_0 + U (\sim 1\text{kHz})$$

$$U = \frac{4\pi\hbar^2 a}{m} \int |\phi(\mathbf{r})|^4 d^3\mathbf{r}$$

BEC



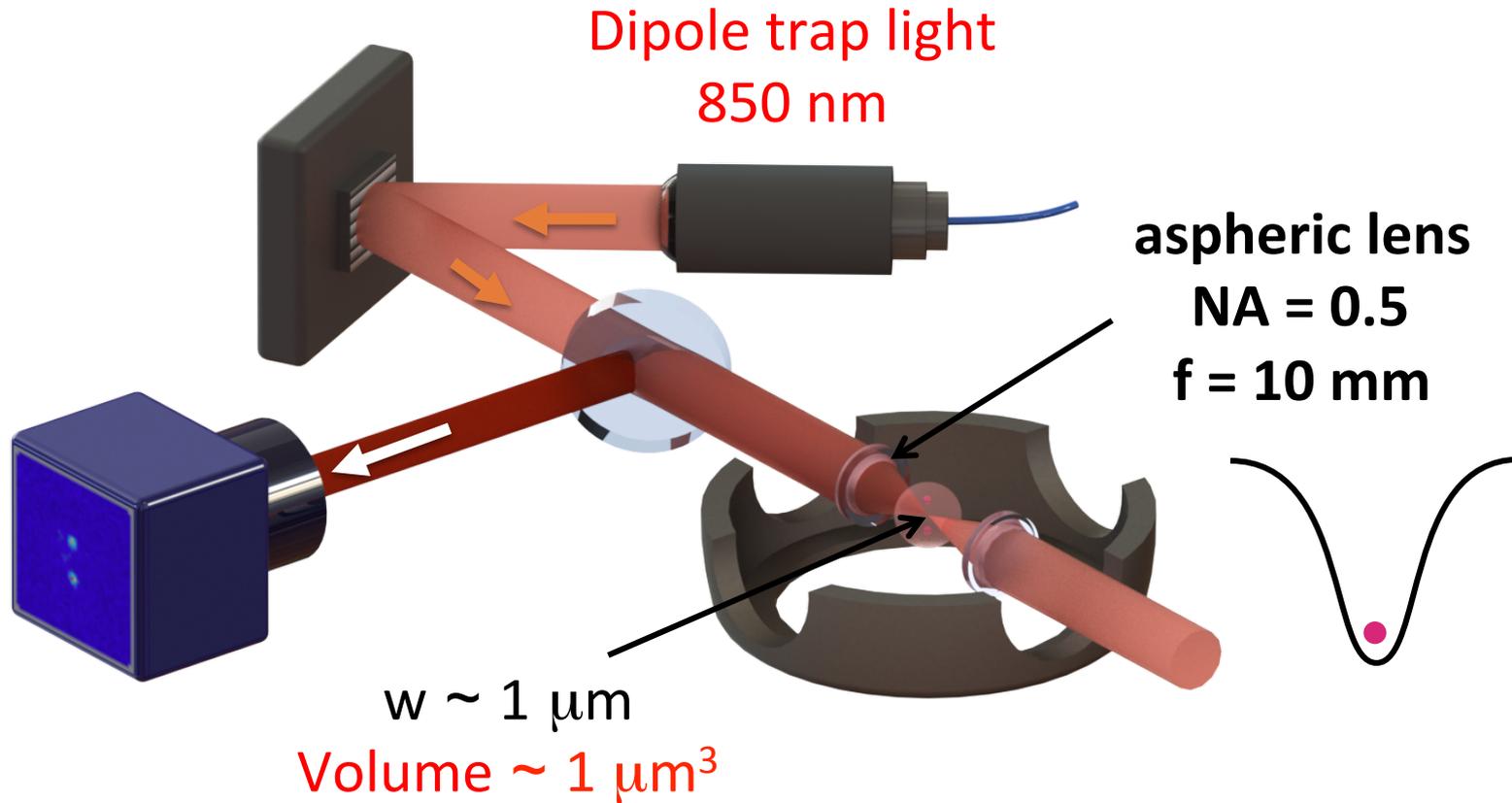
Adiabatic increase:
connects ground-states



Works in 2D and 3D

Microscopic dipole traps (tweezers) for single atoms

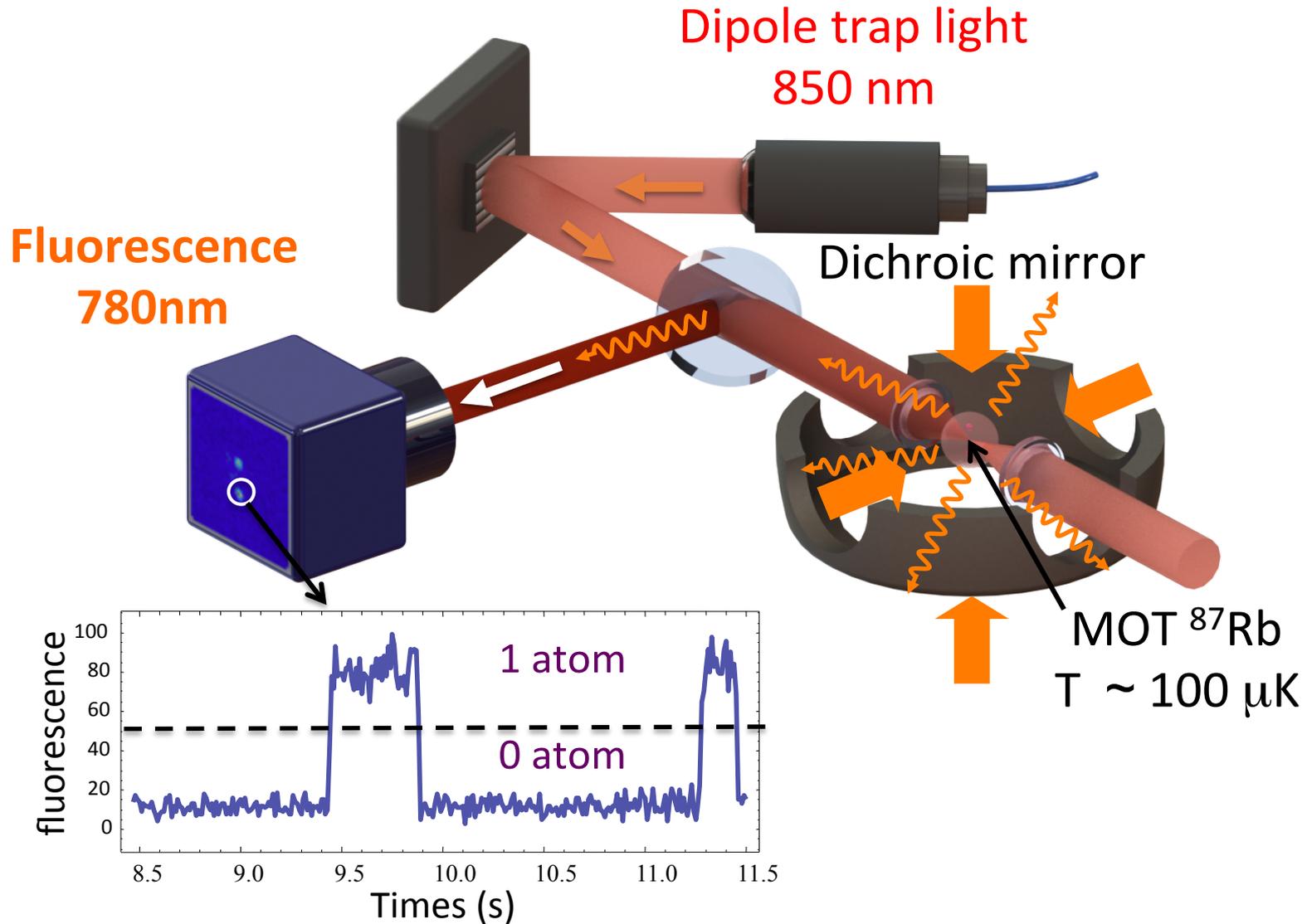
Schlosser, Nature (2001); Sortais, PRA (2007); Nogrette, PRX (2014)



Related works: Darmstadt, Amsterdam, Wisconsin, Harvard, Otago...

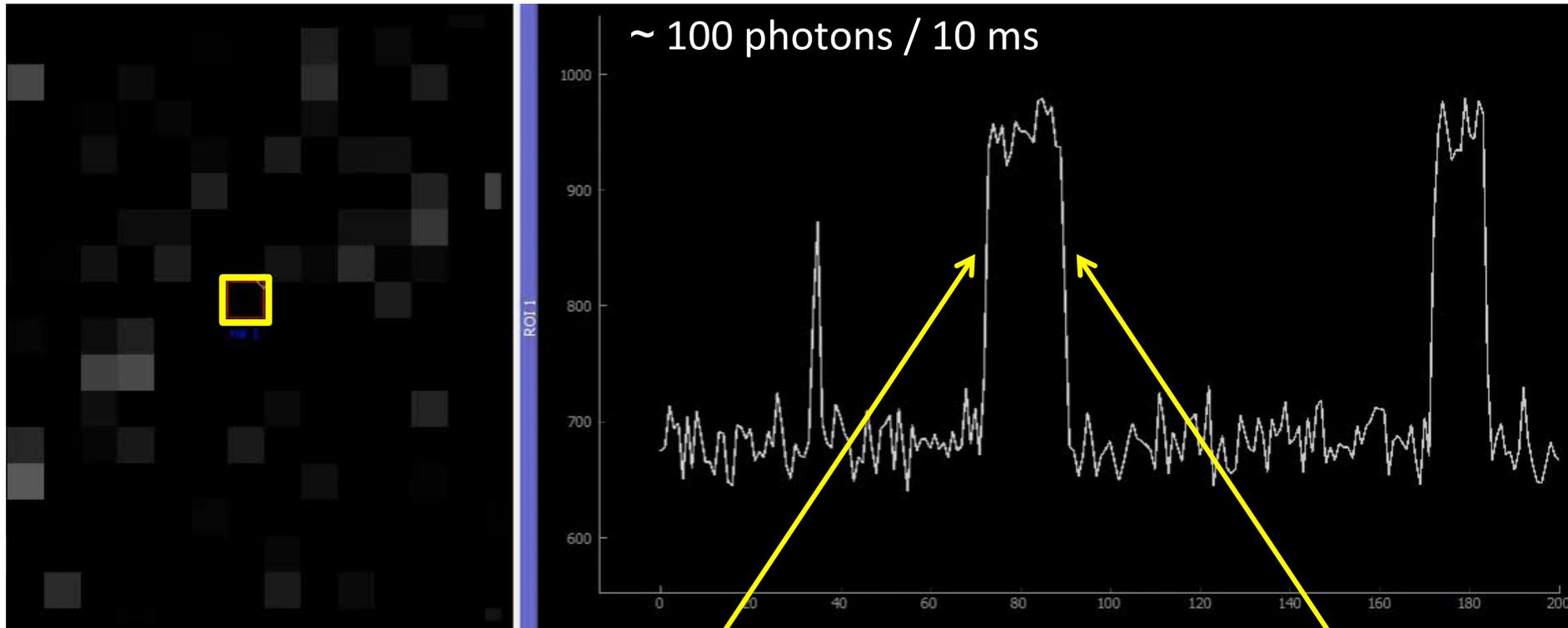
Microscopic dipole traps (tweezers) for single atoms

Schlosser, Nature (2001); Sortais, PRA (2007); Nogrette, PRX (2014)



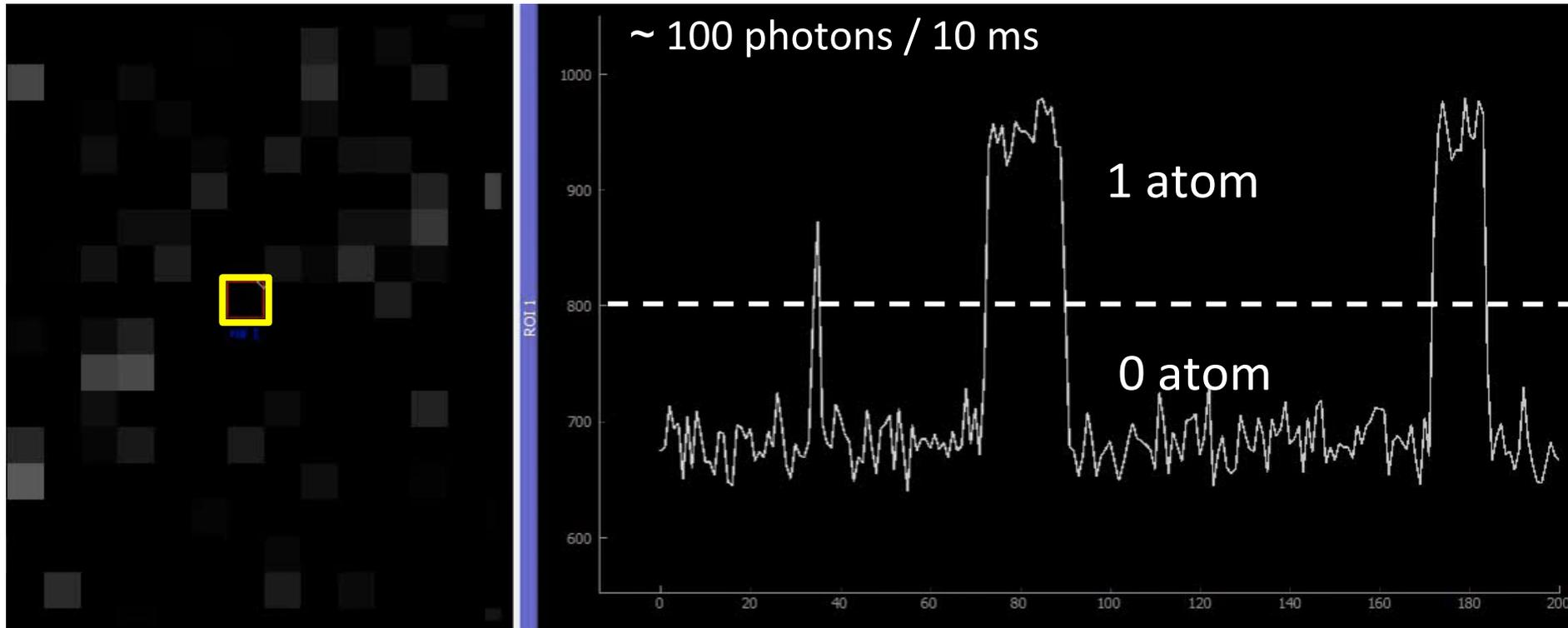
Fast light-assisted collision prevents two atoms at the same time...

Fluorescence @ 780 nm induced by the cooling lasers



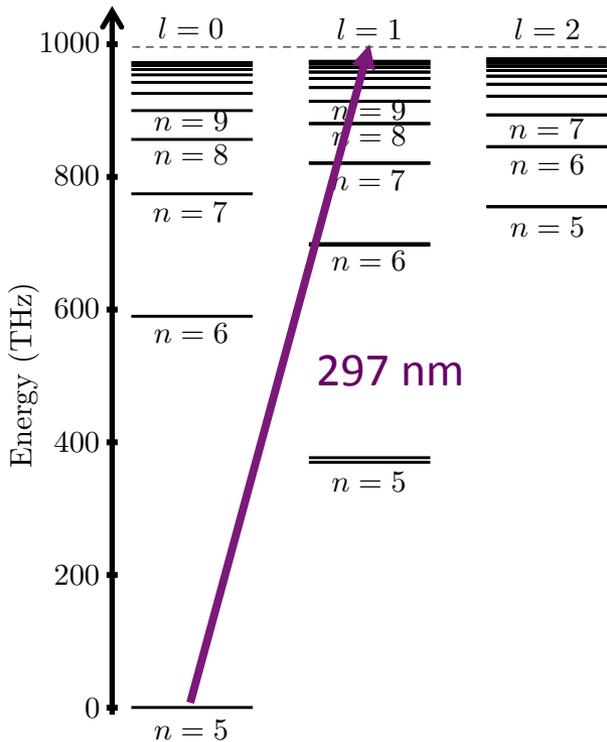
Fast light-assisted collision prevents two atoms at the same time...

Fluorescence @ 780 nm induced by the cooling lasers

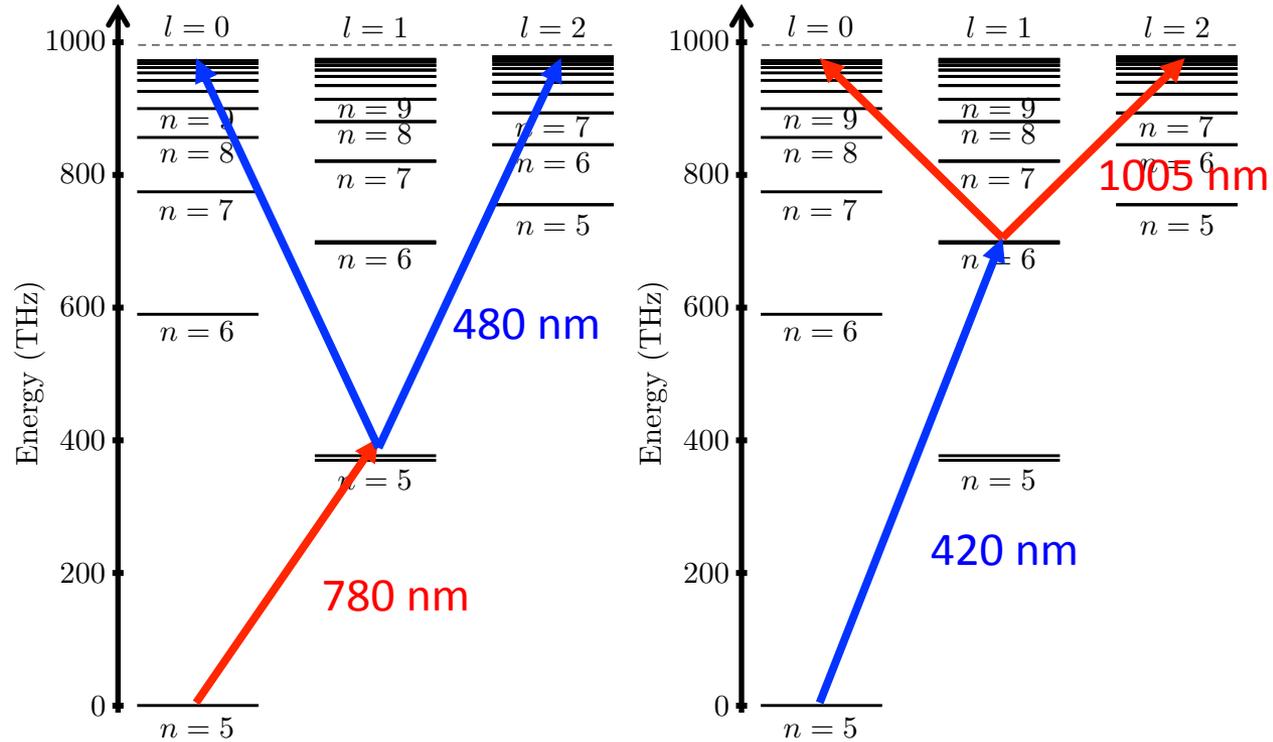


Coherent Rydberg excitation (rubidium)

Single photon (UV)



Two-photon

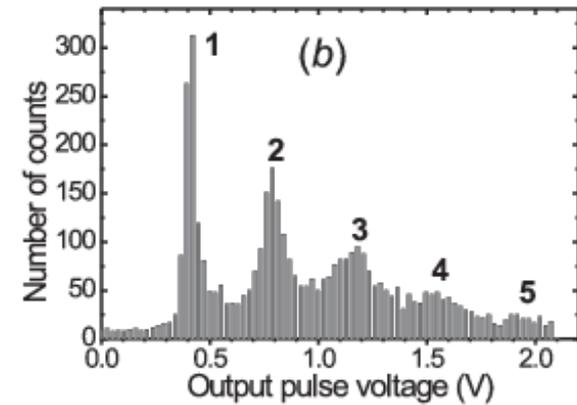
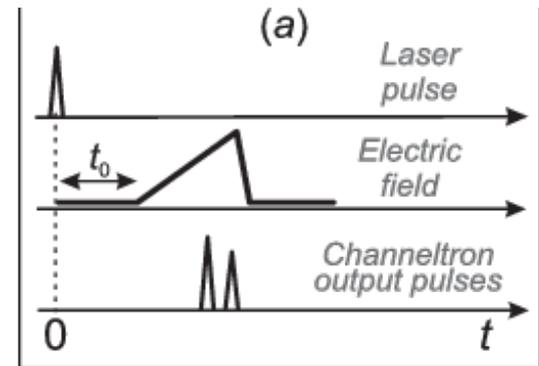
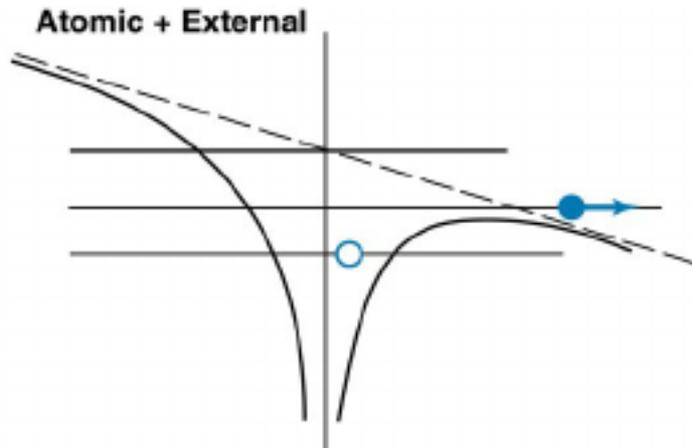
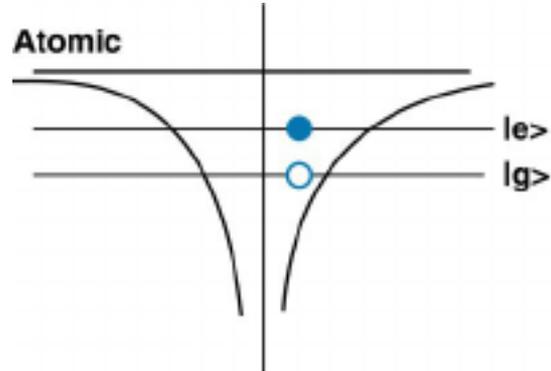


$$\Omega = \frac{\Omega_1 \Omega_2}{2\Delta}$$

$$\text{Light-shift: } \delta_{\text{eff}} = \delta - \left(\frac{|\Omega_R|^2}{4\Delta} - \frac{|\Omega_B|^2}{4\Delta} \right)$$

Electronic detection of Rydberg atoms

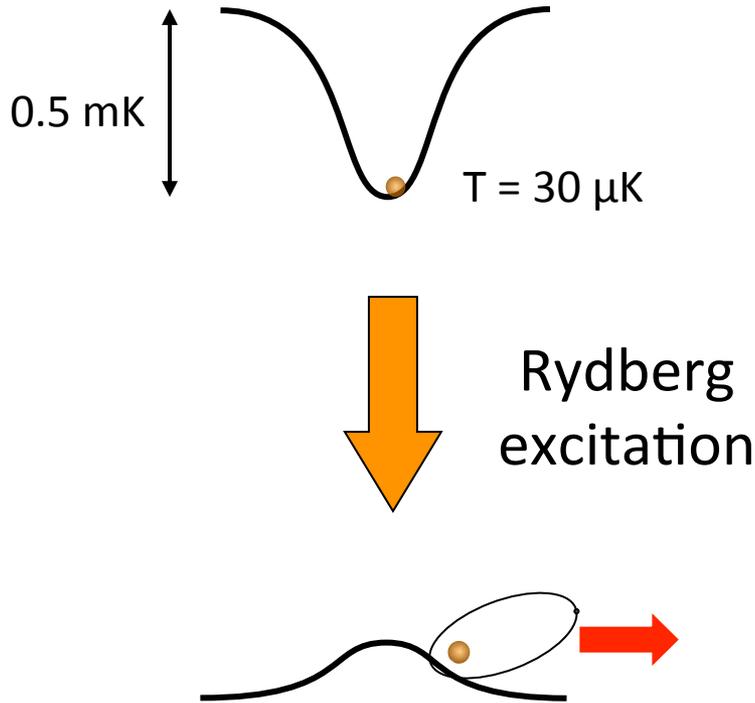
Field ionization



I. Beterov

Detection of Rydberg atoms

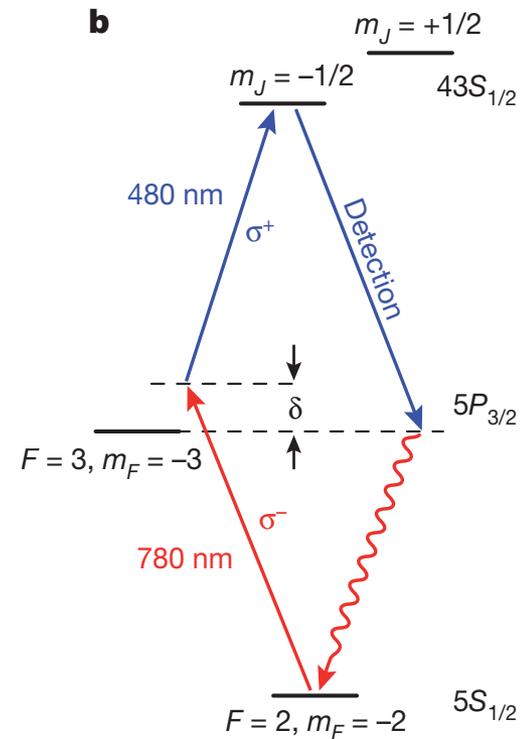
Atom loss



Leave trapping region
(1 μm) in $< 5 \mu\text{sec}$

Efficiency $> 95\%$

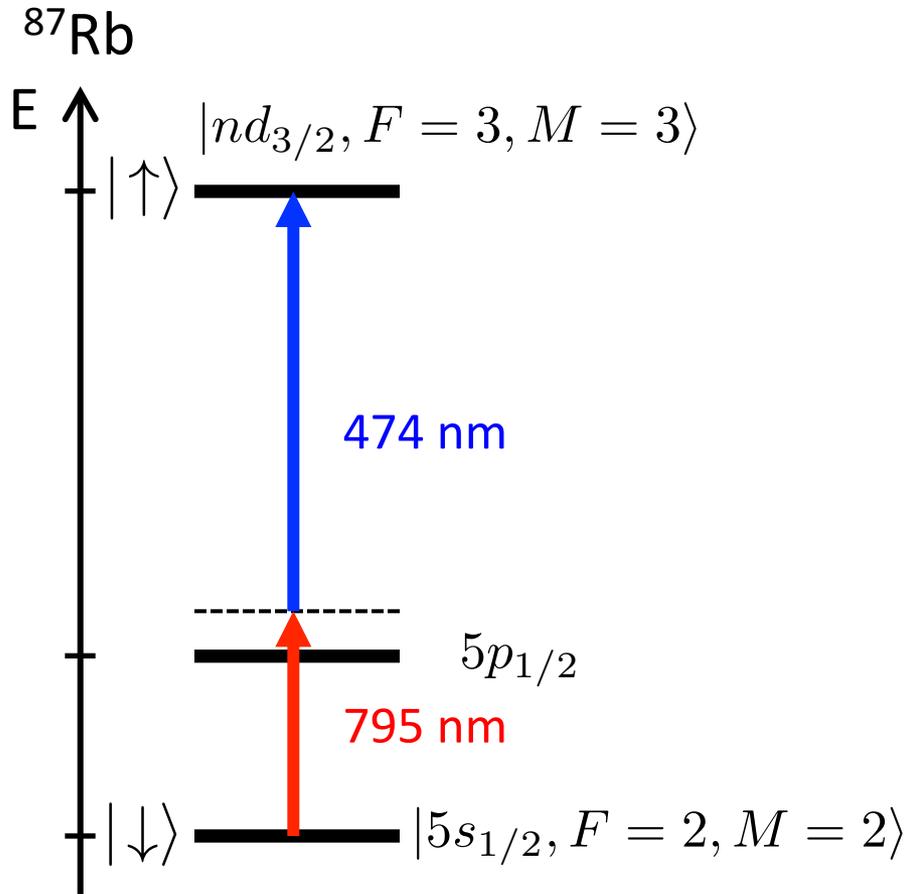
"Optical" detection



Schauss, Nature **491**, 87 (2012)

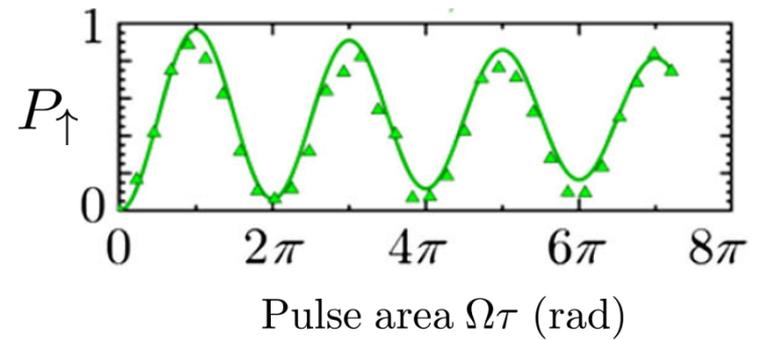
Efficiency $\sim 80\%$

Coherent optical Rydberg excitation ($n = 50 - 100$)



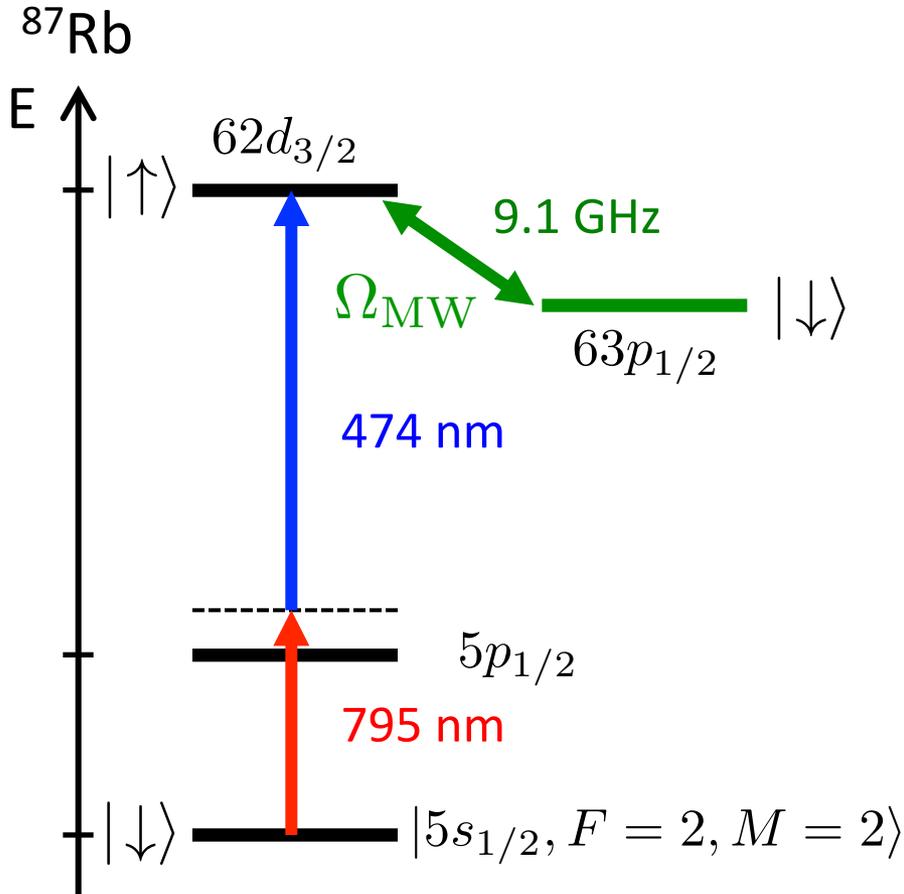
Single atom \Rightarrow repeat 100 times

Optical excitation ($\Omega = 0.5 - 5\text{ MHz}$)



T. A. Johnson *et al.*, PRL **100**, 113003 (2008)
Miroshnychenko, PRA **82**, 023623 (2010)

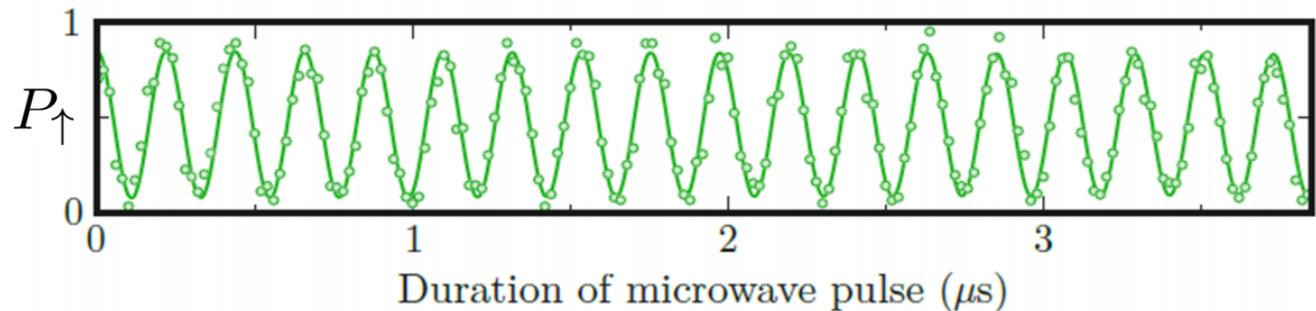
Microwave manipulations ($n = 50 - 100$)



Single atom \Rightarrow repeat 100 times

Microwave transfer

D. Barredo *et al.*,
PRL **114**, 113002 (2015)



Interactions between Rydberg atoms

References:

“Experimental investigations of dipole–dipole interactions between a few Rydberg atoms”, A. Browaeys *et al.*, J. Phys. B 49, 152001 (2016)

“Calculation of Rydberg interaction potentials”, S. Weber *et al.*, J. Phys. B **50**, 133001 (2017)

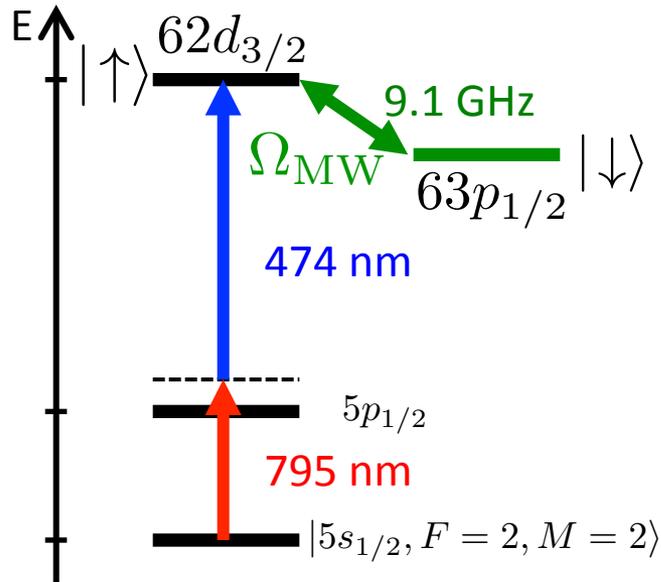
Softwares to calculate interaction energies

S. Weber *et al.*, [arXiv:1612.08053](https://arxiv.org/abs/1612.08053), <https://pairinteraction.github.io>

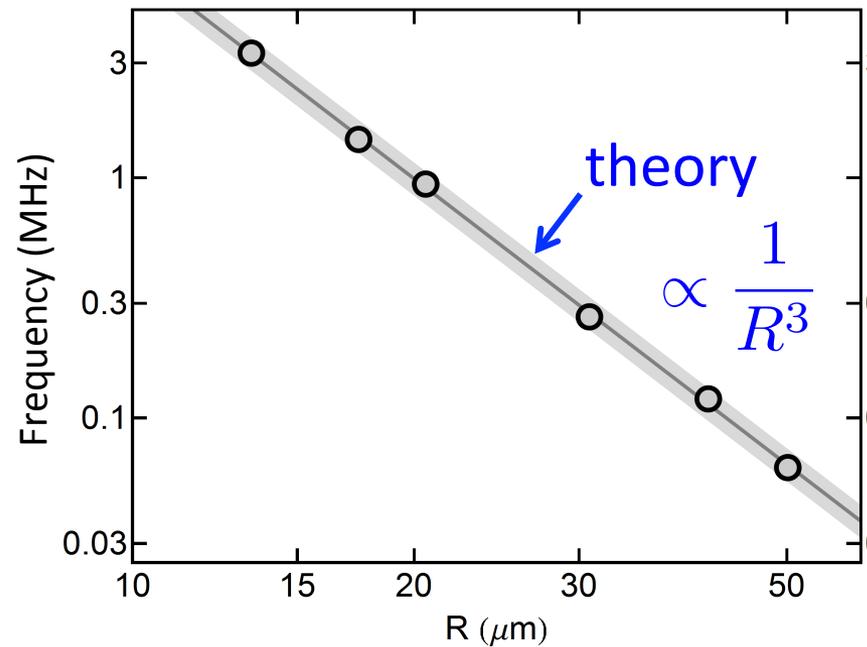
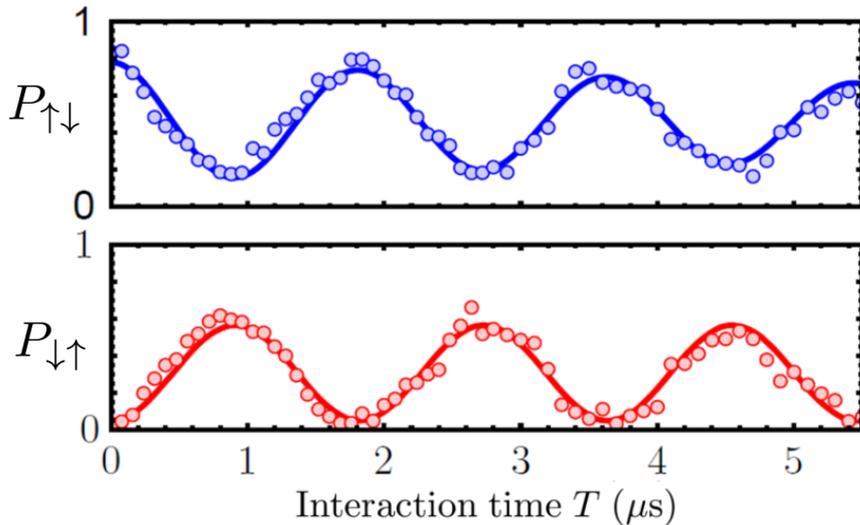
ARC: An open-source library for calculating properties of alkali Rydberg atoms, N. Sibalic *et al.*, [arXiv:1612.05529](https://arxiv.org/abs/1612.05529) (2016)

Observation of spin exchange between 2 atoms ($R = 30 \mu\text{m}$)

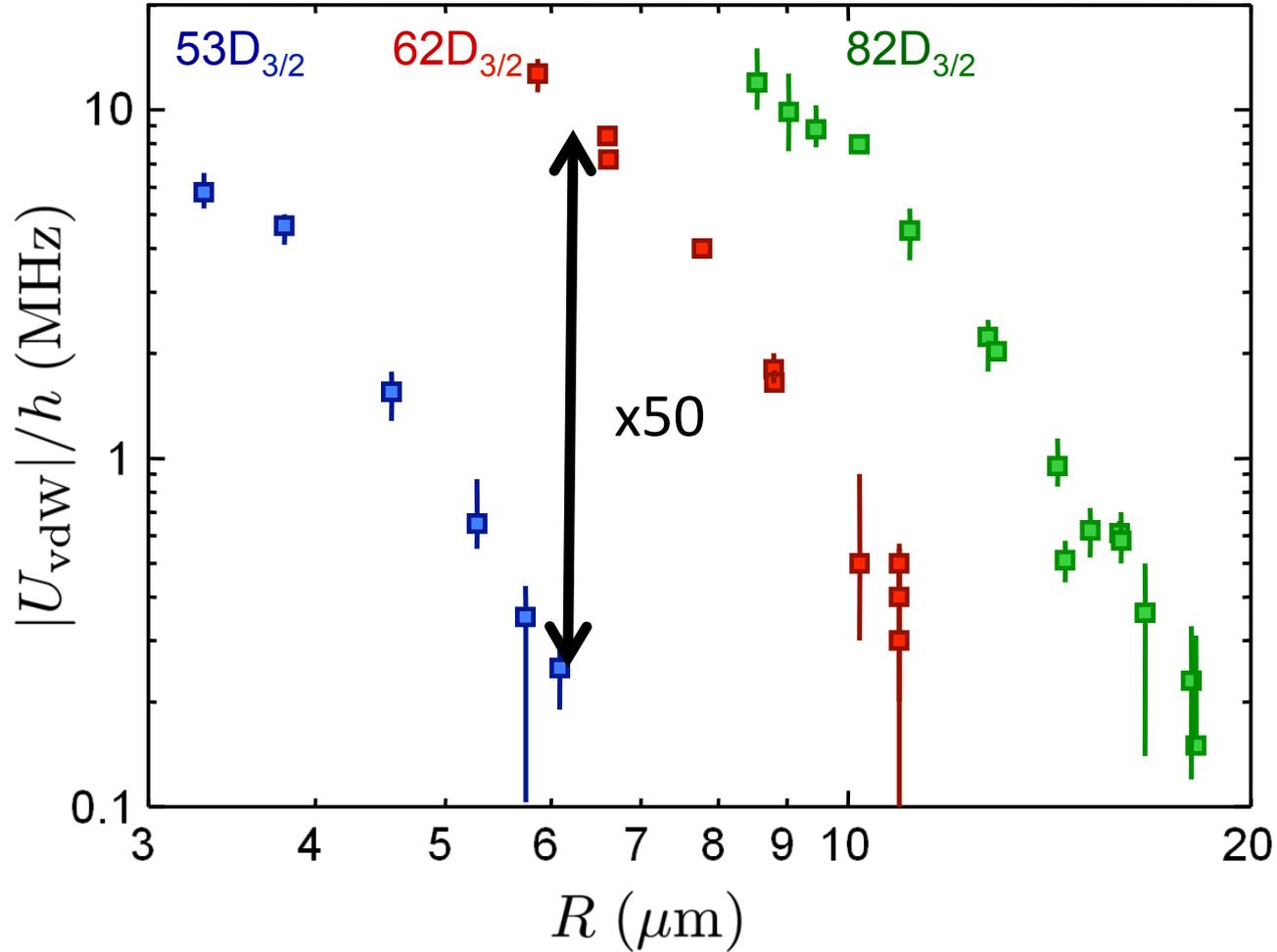
Barredo *et al.*, PRL **114**, 113002 (2015)



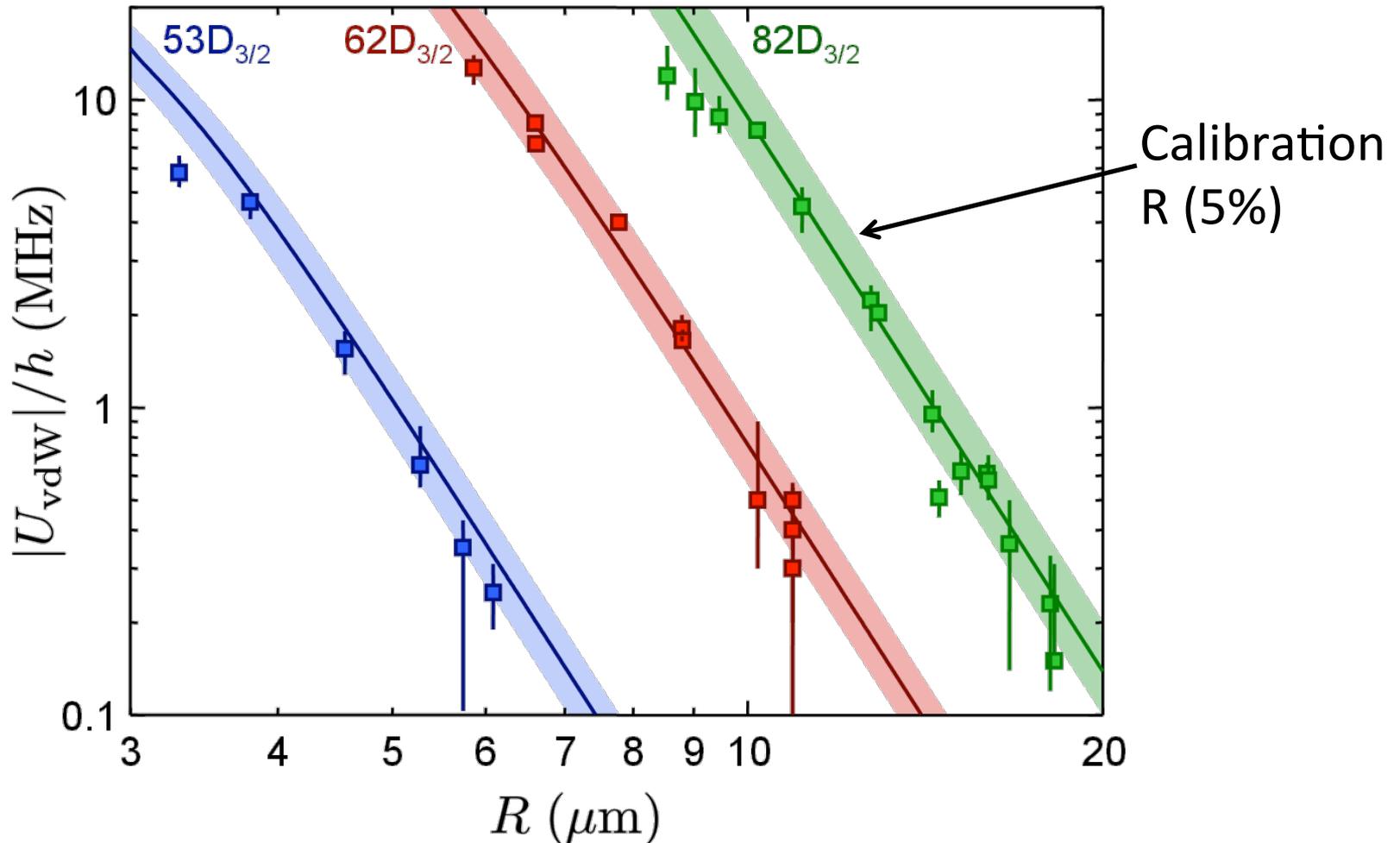
Prepare $|\uparrow\downarrow\rangle$ using **microwaves** + **addressing beam** [Labuhn, PRA **90**, 023415 (2014)]



Measurement of the Van der Waals energy between 2 atoms



Measurement of the Van der Waals energy between 2 atoms

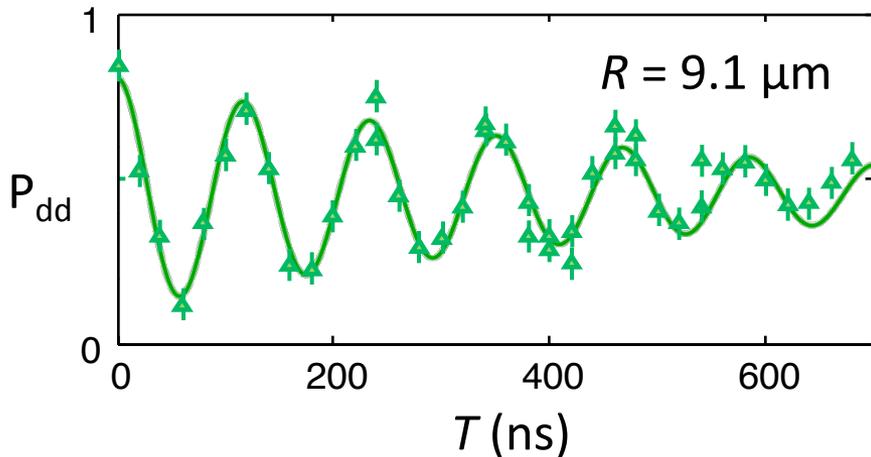
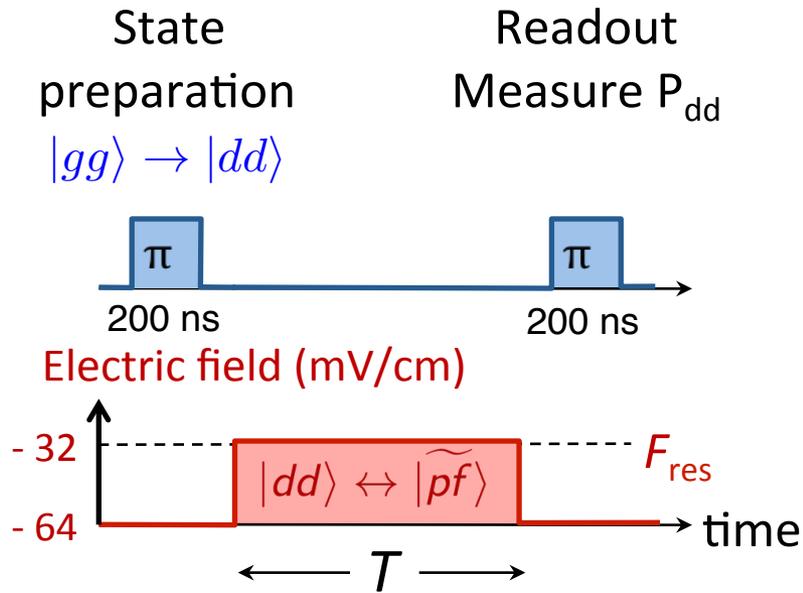


Theory curves: direct diagonalization (dipole-dipole interaction)

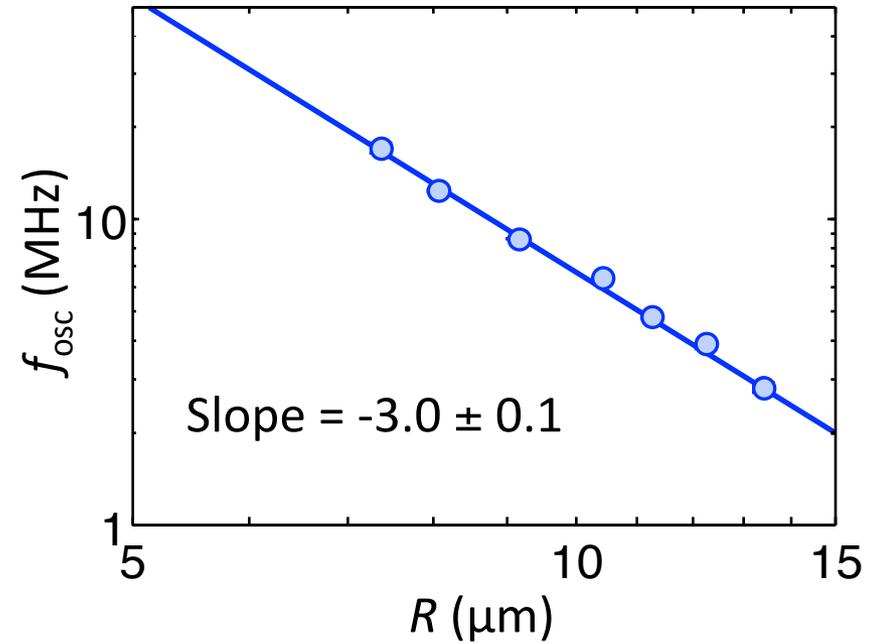
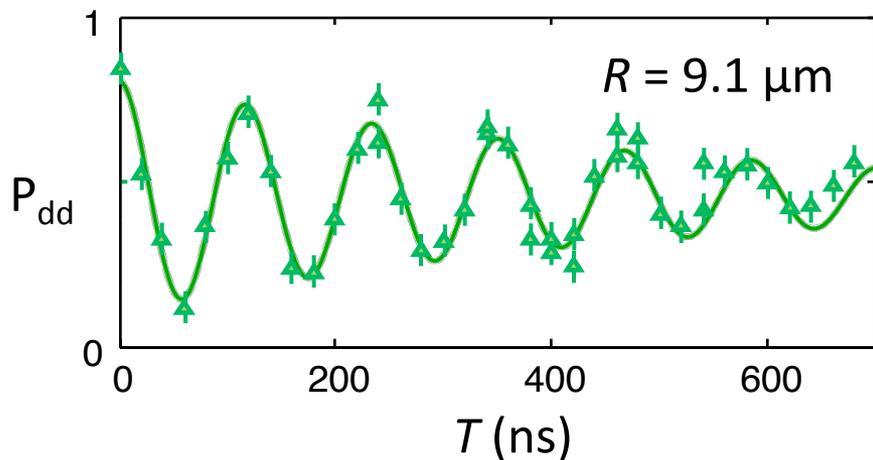
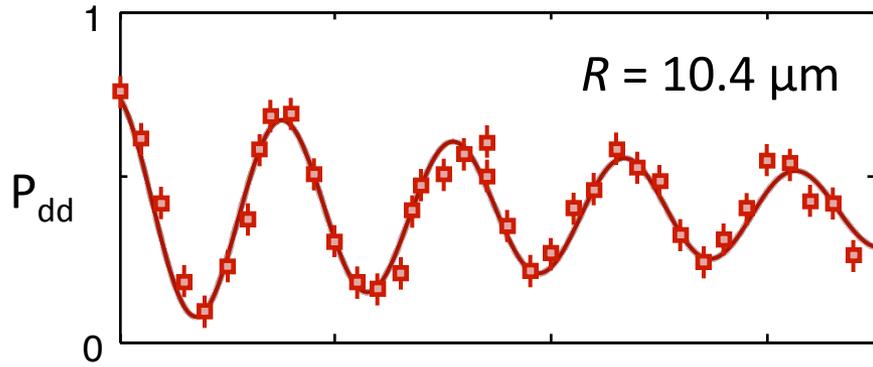
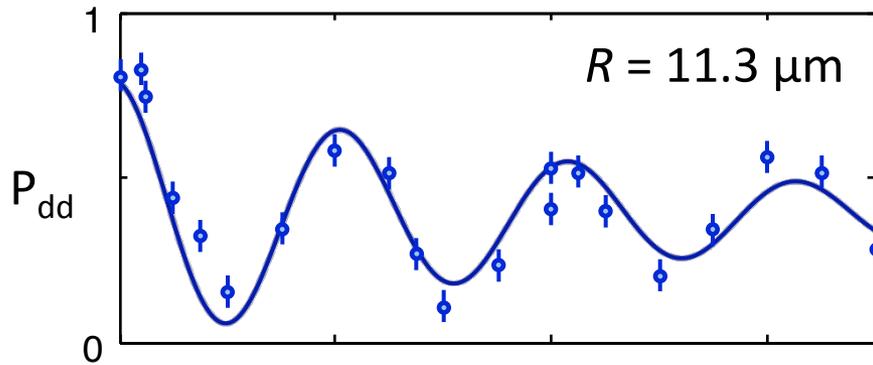
No adjustable parameter!

Béguin *et al.*, Phys. Rev. Lett. **110** 263201 (2013)

Förster resonance: electrically-tuned interaction



Förster resonance: electrically-tuned interaction



$$C_{3,\text{exp}} = 2.39 \pm 0.03 \text{ GHz} \cdot \mu\text{m}^3$$

$$C_{3,\text{th}} = 2.54 \text{ GHz} \cdot \mu\text{m}^3$$

Summary of interaction between Rydberg atoms

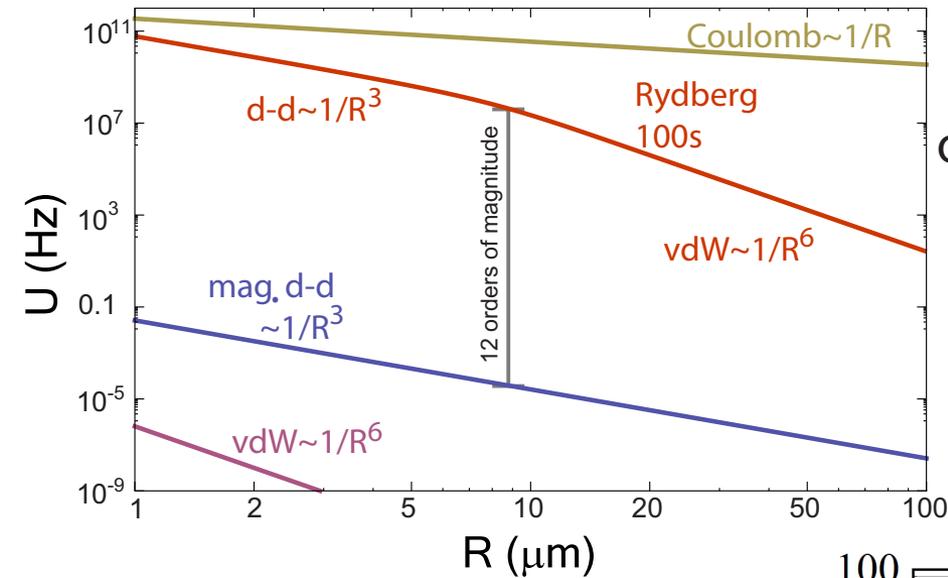
REVIEWS OF MODERN PHYSICS, VOLUME 82, JULY–SEPTEMBER 2010

Quantum information with Rydberg atoms

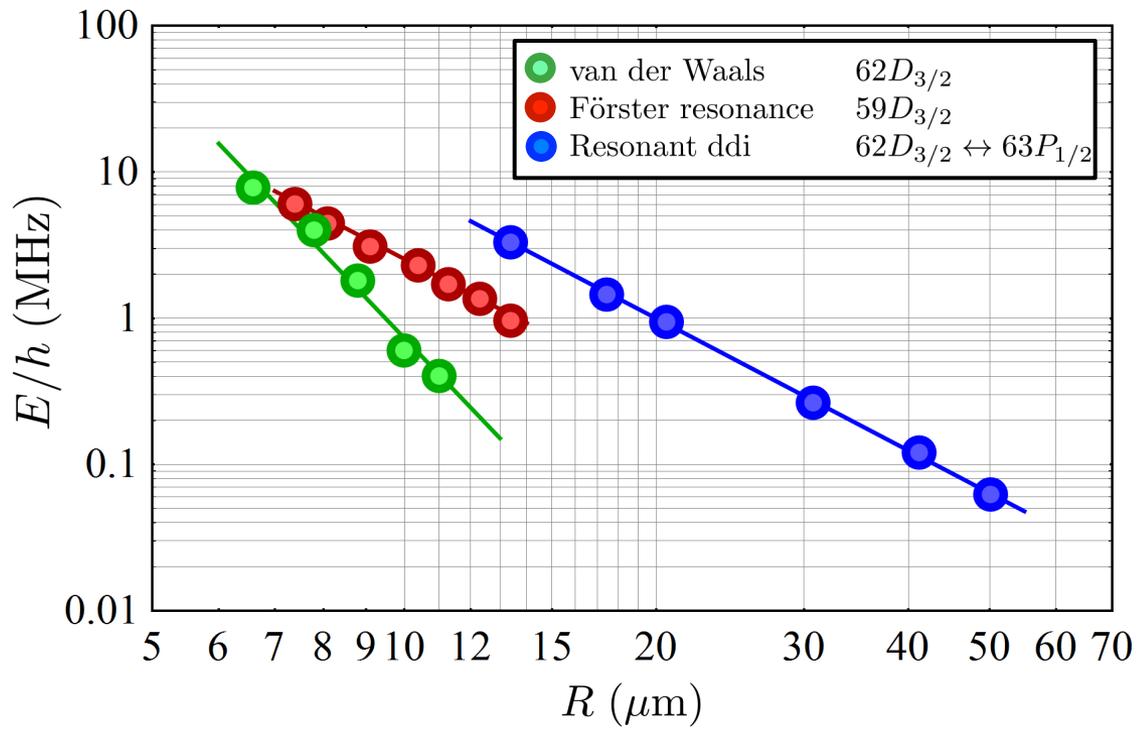
M. Saffman and T. G. Walker

Department of Physics, University of Wisconsin, 1150 University Avenue, Madison, Wisconsin 53706, USA

K. Mølmer



Summary of Palaiseau experiments (2013-2015) using **individual** atoms



Outline

Lecture 1: Rydberg atoms and their interaction

Lecture 2: Rydberg Blockade and application to QIP

Lecture 3: Quantum simulation & Quantum Optics with Rydbergs

Blockade with 2 individual atoms

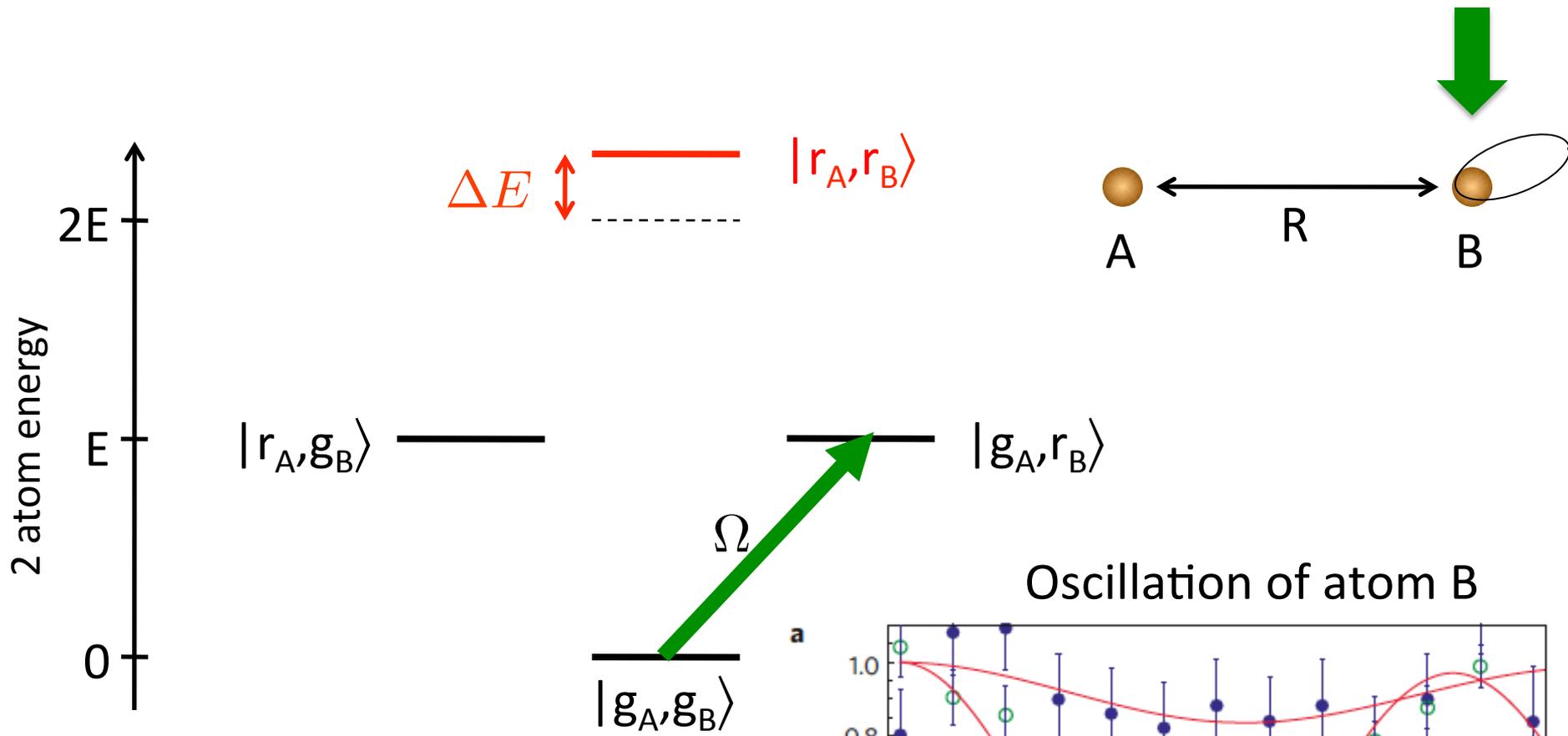
Application to gates and entanglement

References:

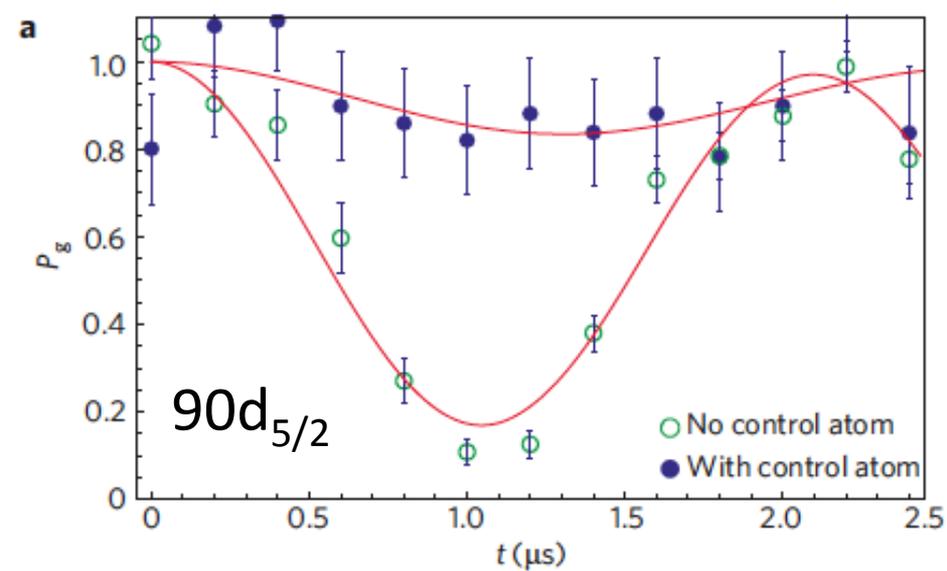
“Quantum Information with Rydberg atoms”, M. Saffman, T. Walker, K. Moelmer, Rev. Mod. Phys. **82**, 2313 (2010)

“Quantum computing with atomic qubits and Rydberg interactions: progress and challenges”, M. Saffmann., J. Phys. B **49**, 202001 (2017)

Rydberg blockade: “addressable” version (U. Wisconsin)

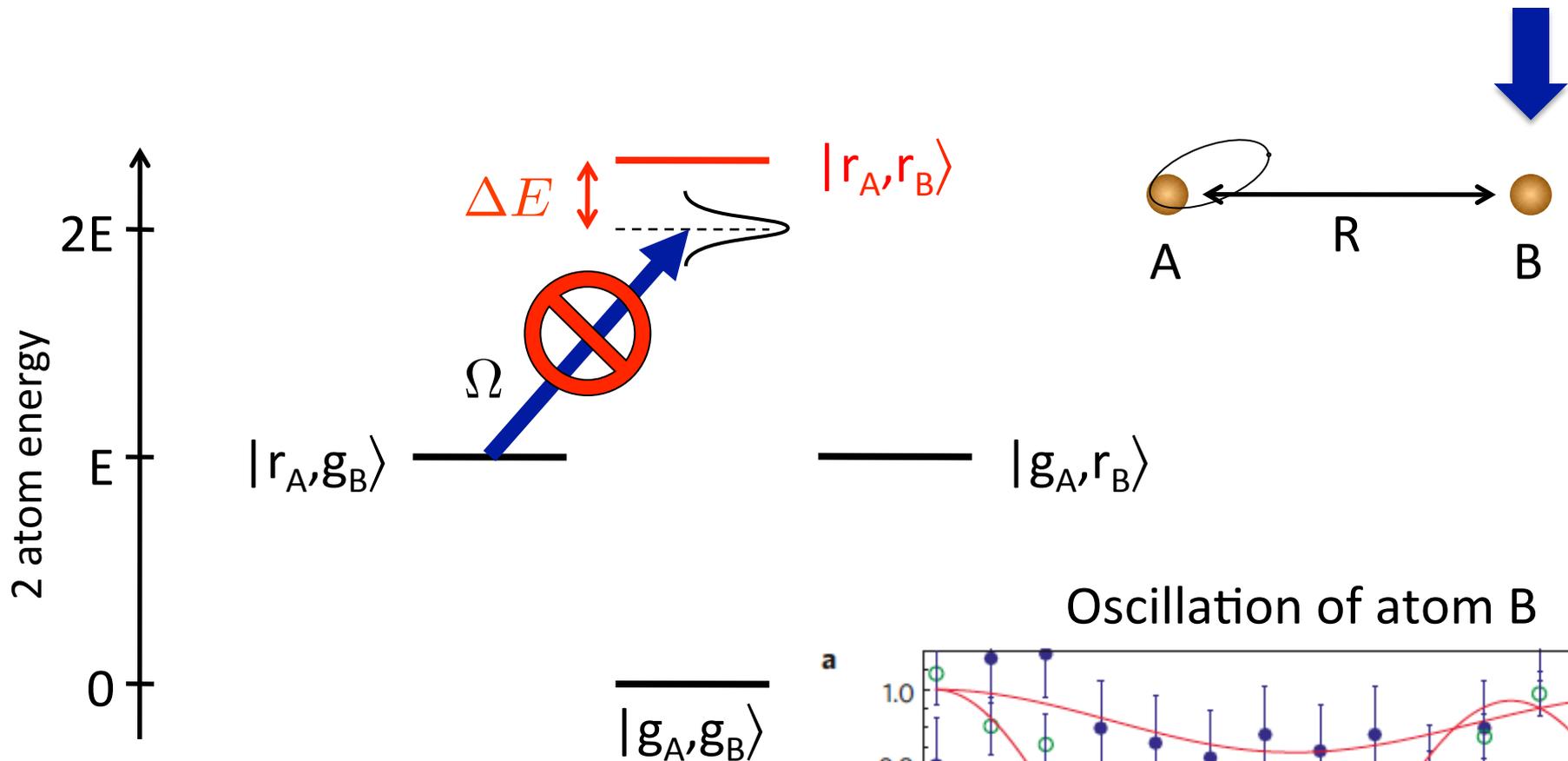


Oscillation of atom B

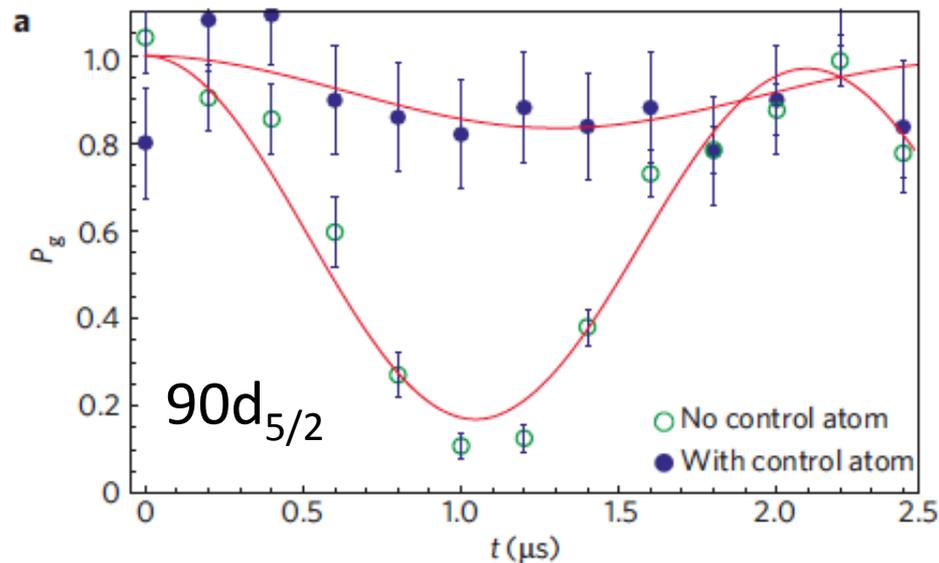


E. Urban *et al.*, *Nat. Phys.* **5**, 110 (2009)

Rydberg blockade: “addressable” version (U. Wisconsin)

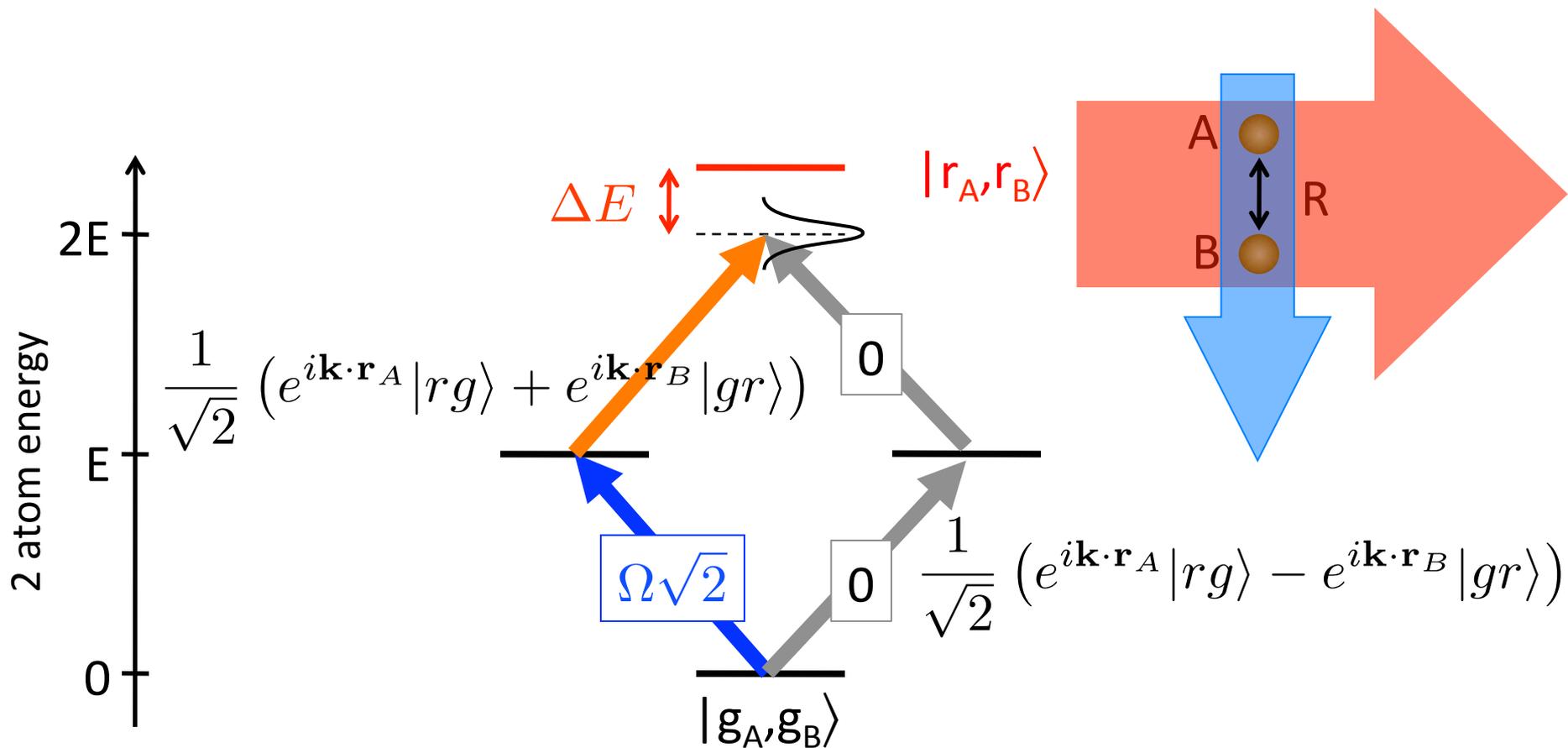


Oscillation of atom B

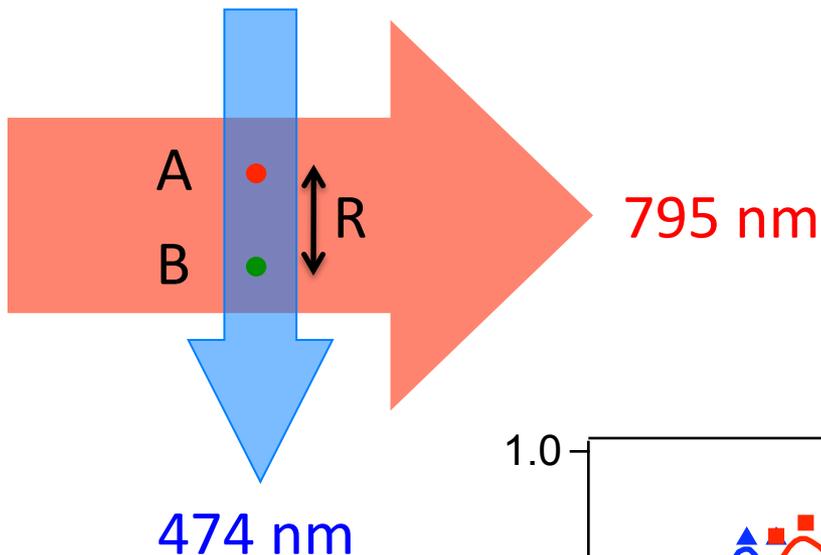


E. Urban *et al.*, *Nat. Phys.* **5**, 110 (2009)

Rydberg blockade: collective excitation (IO Palaiseau)



Rydberg blockade: collective excitation (IO Palaiseau)



Exc. proba atom A only

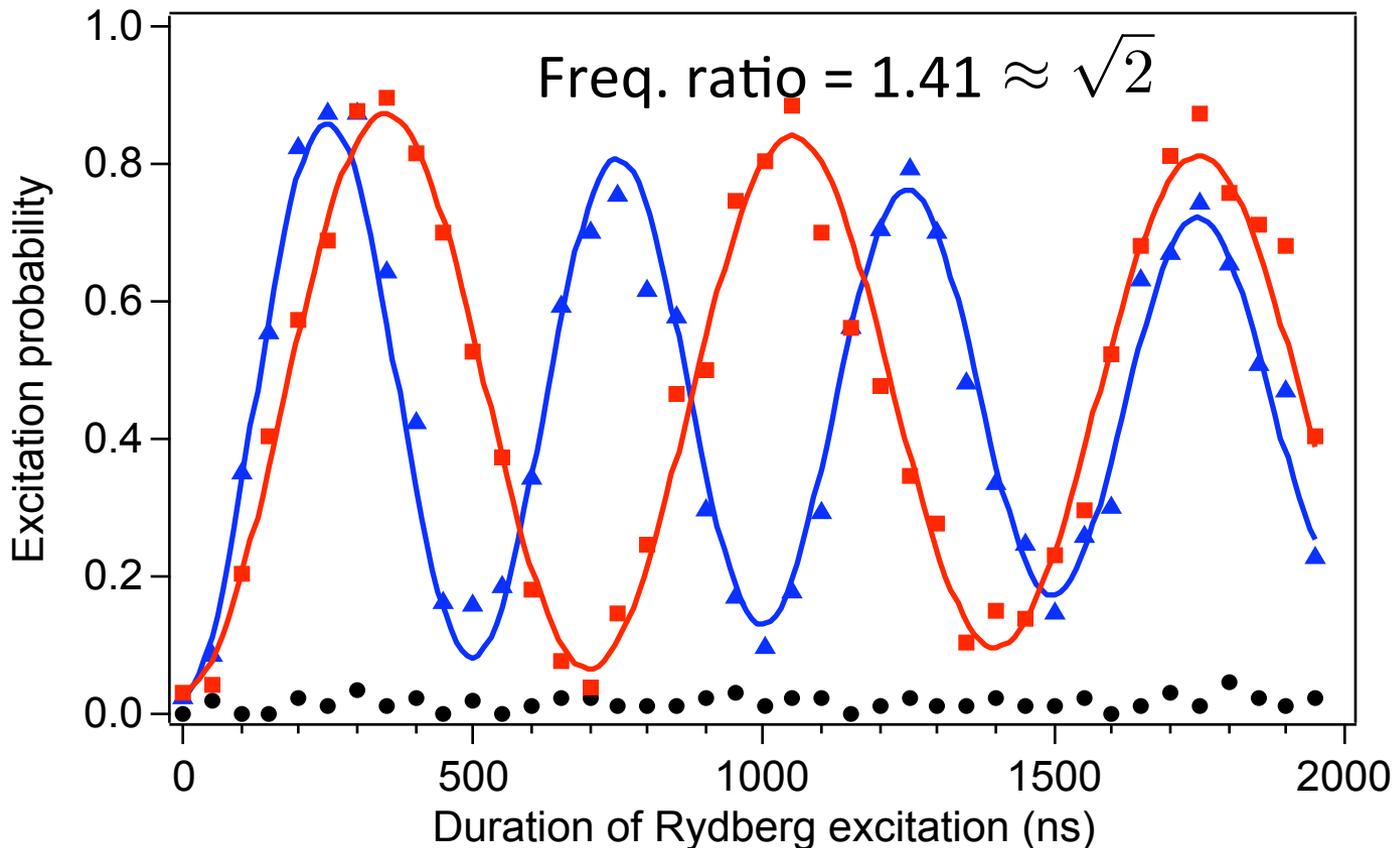
Exc. proba atom A & B

Exc. proba atom A **OR** B

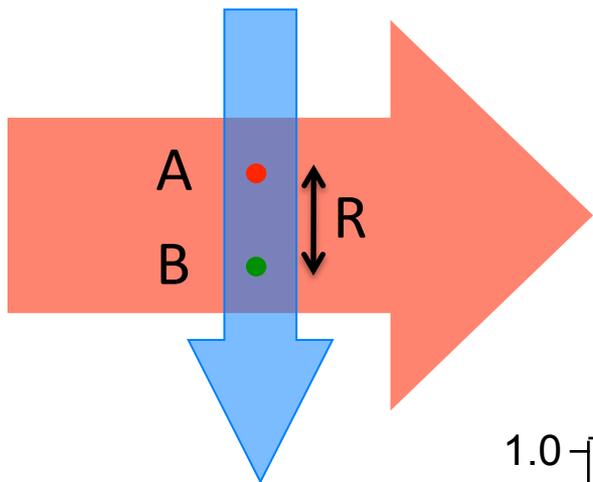
$58d_{3/2}$

$R = 5 \mu\text{m}$

$\Delta E_{\text{theo}} = 25 \text{ MHz}$



Rydberg blockade: collective excitation (IO Palaiseau)



Note: $\phi = \mathbf{k}_{\text{exc}} \cdot (\mathbf{r}_A - \mathbf{r}_B)$

$$\frac{1}{\sqrt{2}} (|r, g\rangle + e^{i\phi} |g, r\rangle)$$

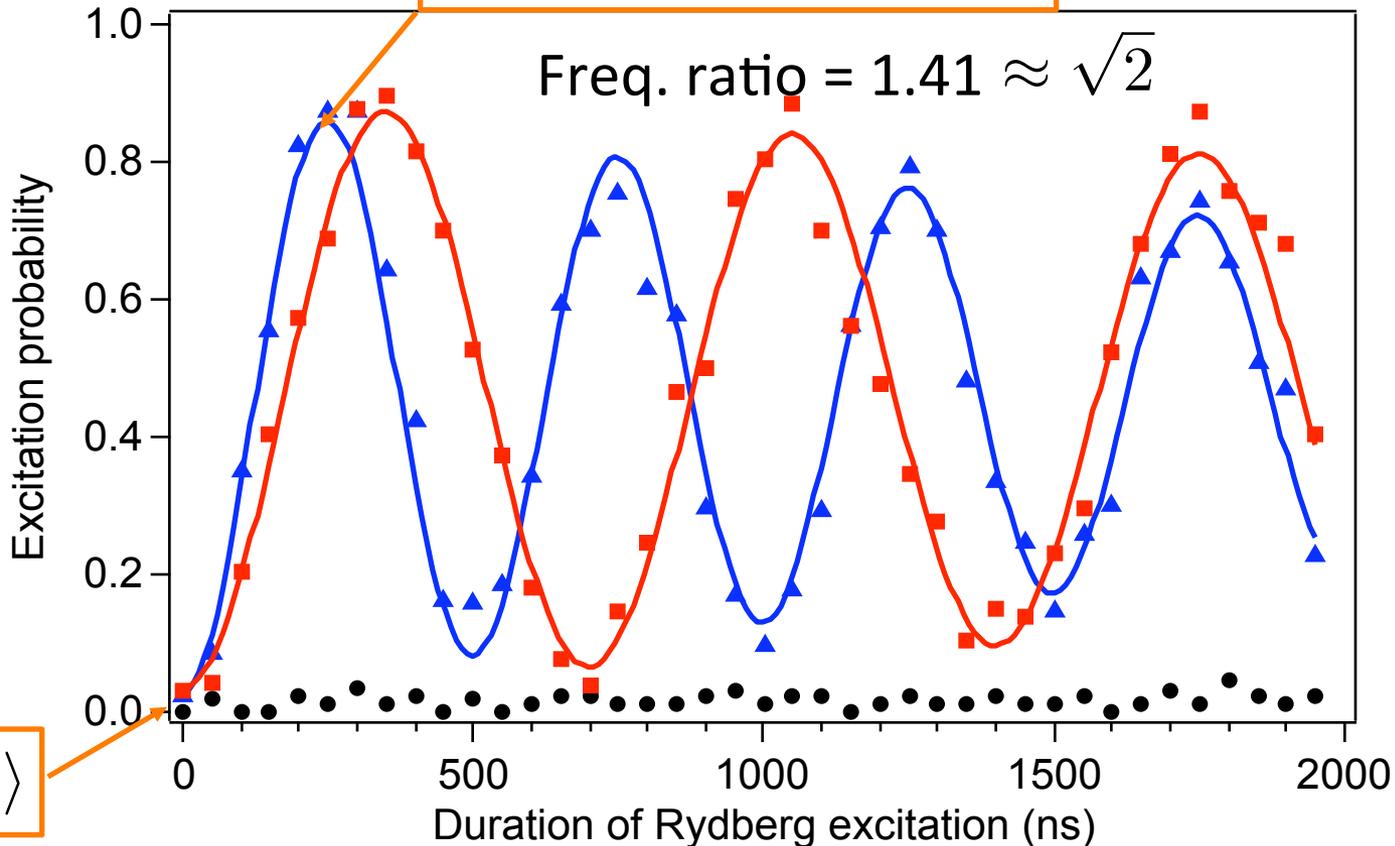
474 nm

795 nm

$58d_{3/2}$

$R = 5 \mu\text{m}$

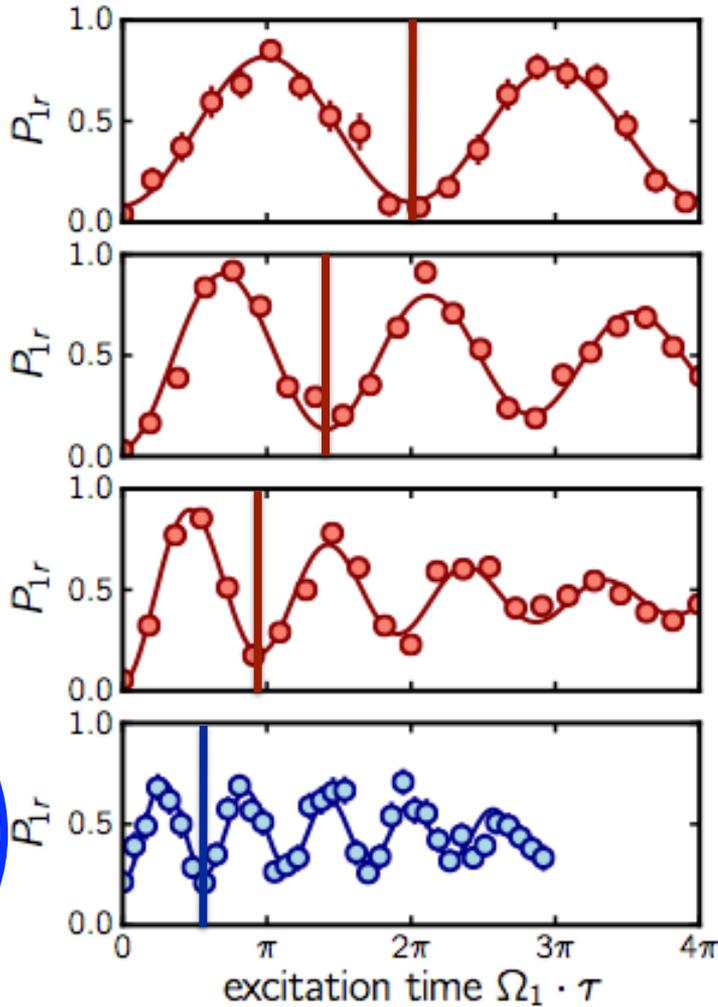
$\Delta E_{\text{theo}} = 25 \text{ MHz}$



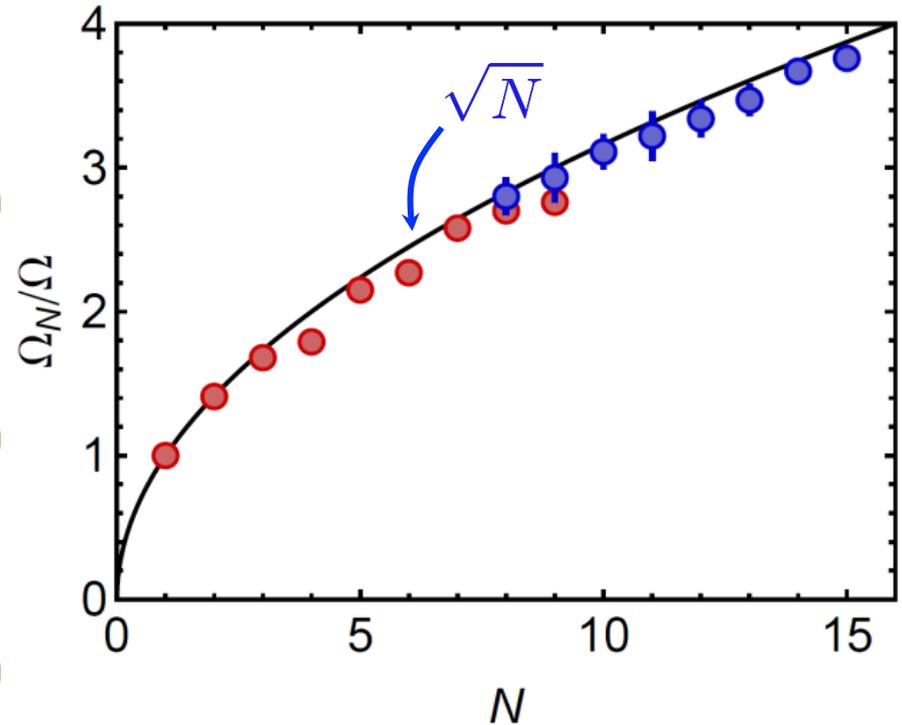
Collective excitation and Rydberg blockade

- trap
- atom

1 Rydberg excitation

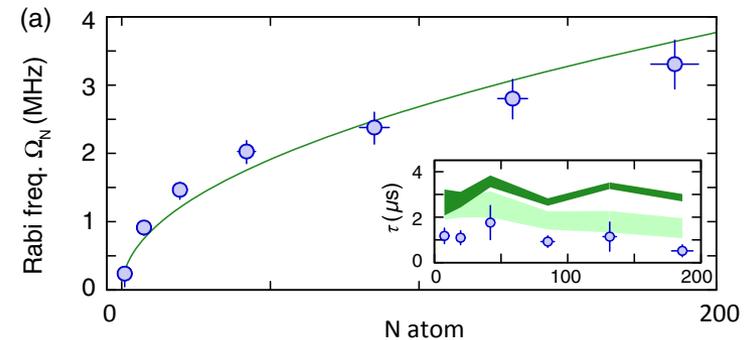
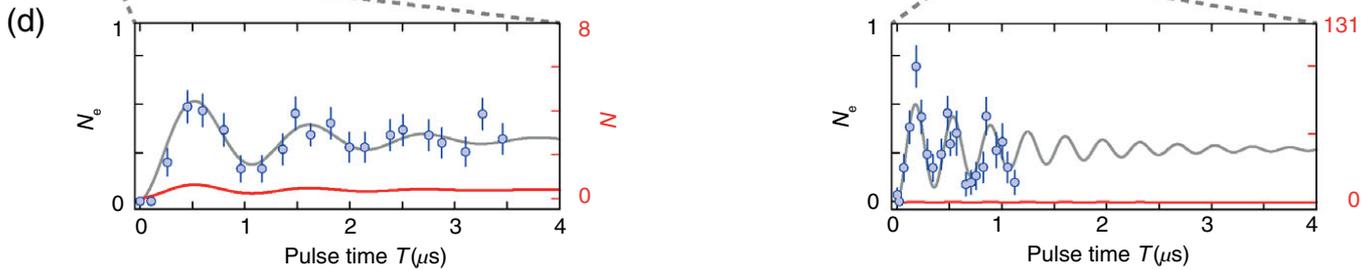
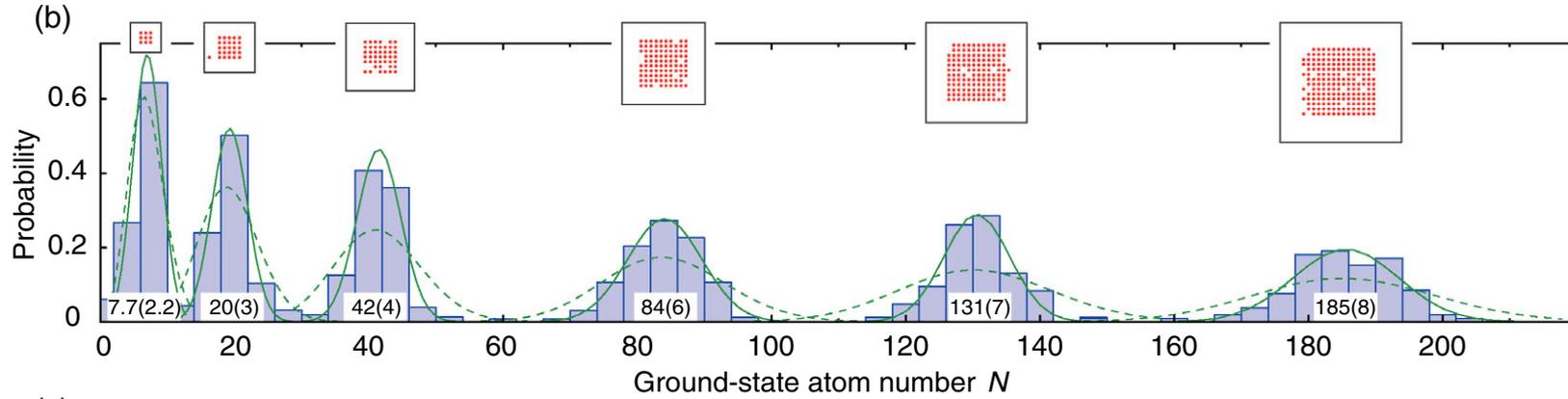


● 100 realizations



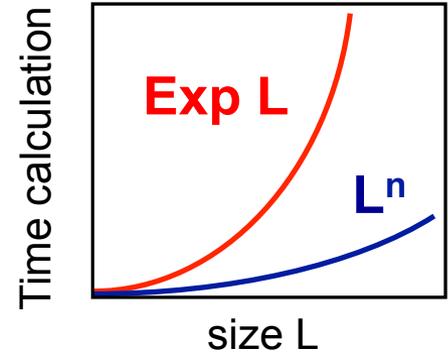
Labuhn *et al.*, Nature (2016)

Collective excitation in sub-poissonian ensemble (MPQ, Garching)



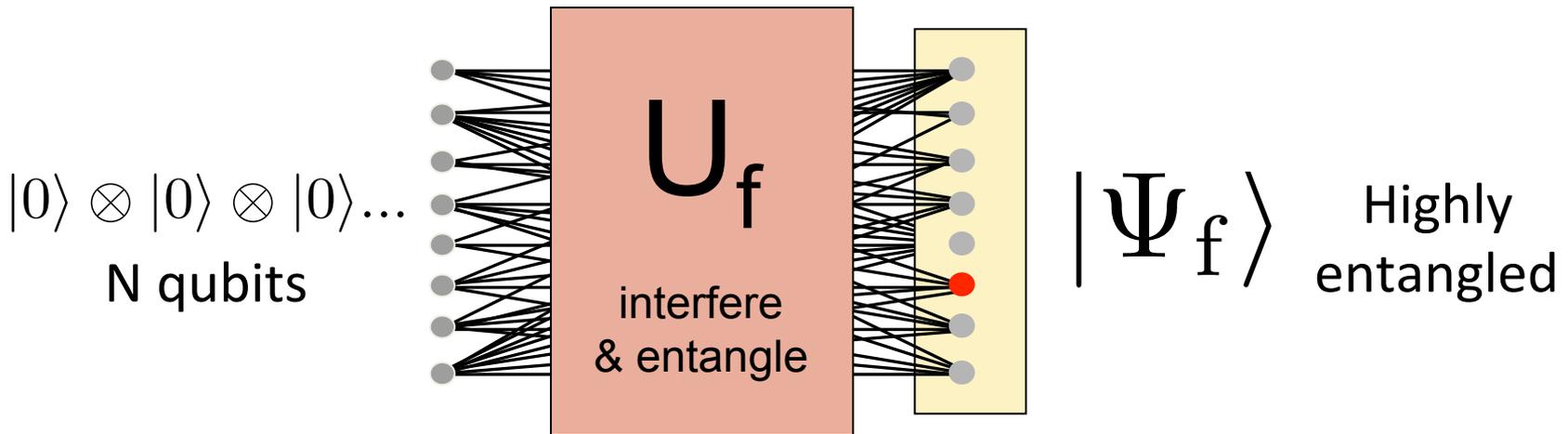
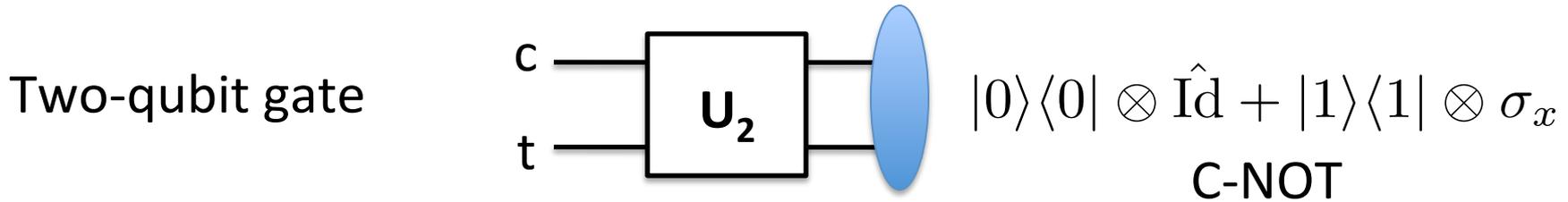
Entanglement = resource for quantum computation

Help solving “hard” problems: factoring (Shor)
searching (Grover)



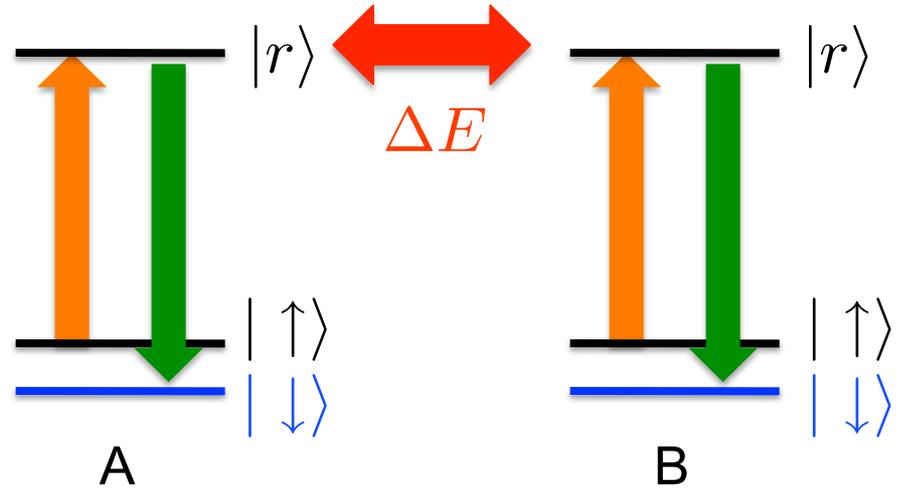
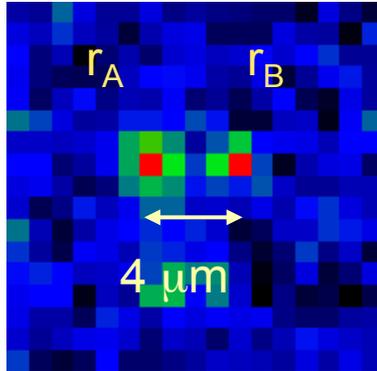
Encode information on a **qubit**: $|0\rangle$, $|1\rangle$

Elementary bricks (“circuit” approach):



Entanglement of two atoms using the Rydberg blockade

Collective excitation version



$$|\uparrow, \uparrow\rangle \xrightarrow{\text{orange}} \frac{1}{\sqrt{2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}_A} |r, \uparrow\rangle + e^{i\mathbf{k}\cdot\mathbf{r}_B} |\uparrow, r\rangle \right) \quad k = k_R + k_B$$

$$\xrightarrow{\text{green}} \frac{1}{\sqrt{2}} \left(e^{i(\mathbf{k}\cdot\mathbf{r}_A - \mathbf{k}'\cdot\mathbf{r}'_A)} |\downarrow, \uparrow\rangle + e^{i(\mathbf{k}\cdot\mathbf{r}_B - \mathbf{k}'\cdot\mathbf{r}'_B)} |\uparrow, \downarrow\rangle \right)$$

$$\Rightarrow \frac{1}{\sqrt{2}} (|\downarrow, \uparrow\rangle + e^{i\phi} |\uparrow, \downarrow\rangle) \quad \text{with } \phi = \mathbf{k} \cdot (\delta\mathbf{r}_A - \delta\mathbf{r}_B) \quad (k \approx k')$$

If atomic motion frozen $\Rightarrow \delta r_A \approx \delta r_B \approx 0$

Analyzing entanglement

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|\downarrow, \uparrow\rangle + |\uparrow, \downarrow\rangle)$$

Measure the density matrix: $\rho_{\text{exp}} = F|\psi_+\rangle\langle\psi_+| + \rho_{\text{junk}}$

$$\hat{\rho} = \begin{pmatrix} P_{\downarrow\downarrow} & a & b & c \\ a^* & P_{\downarrow\uparrow} & \rho_{\uparrow\downarrow, \downarrow\uparrow} & d \\ b^* & \rho_{\uparrow\downarrow, \downarrow\uparrow}^* & P_{\uparrow\downarrow} & e \\ c^* & d^* & e^* & P_{\uparrow\uparrow} \end{pmatrix} \begin{matrix} |\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle \end{matrix}$$

Extract the fidelity: $F = \langle\psi_+|\hat{\rho}|\psi_+\rangle$

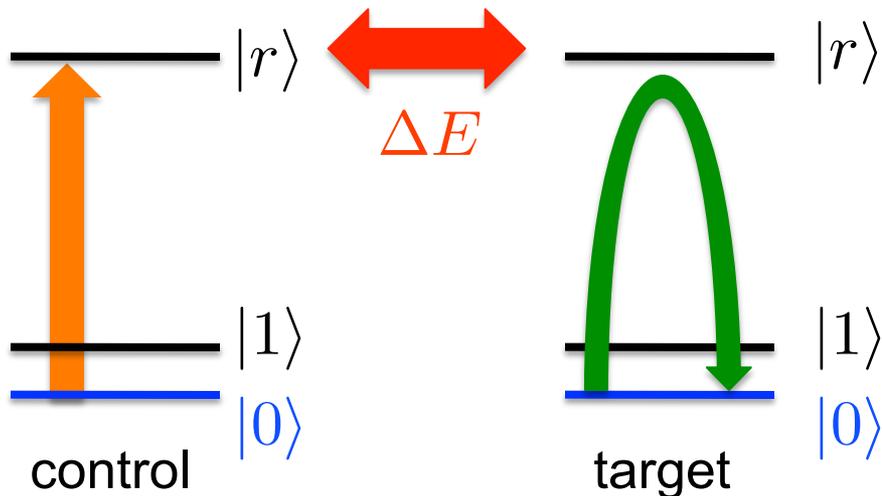
$$F = \frac{1}{2}(P_{\downarrow\uparrow} + P_{\uparrow\downarrow} + 2\Re(\rho_{\downarrow\uparrow, \uparrow\downarrow}))$$

$$F_{\text{pairs}} = 0.75 \pm 0.07$$

Details in Gaëtan *et al.*,
NJP **12**, 065040 (2010)

Quantum gate using Rydberg blockade

D. Jaksch *et al.*, PRL **85**, 2208 (2000)



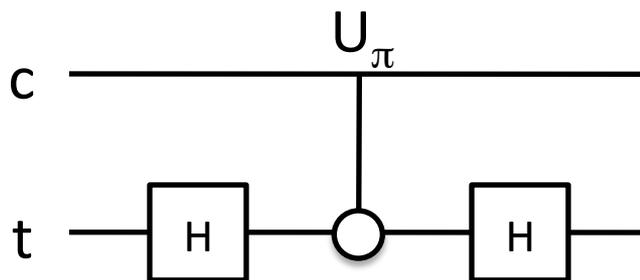
Sequence:
 $\pi_A - 2\pi_B - \pi_A$

Table of truth:

11	→	11	→	11	→	11
10	→	10	→	-10	→	-10
01	→	r1	→	r1	→	-01
00	→	r0	→	r0	→	-00

Blockade

From π -gate to CNOT



11	→	10
10	→	11
01	→	01
00	→	00

The CNOT gate at uni. Wisconsin (1)

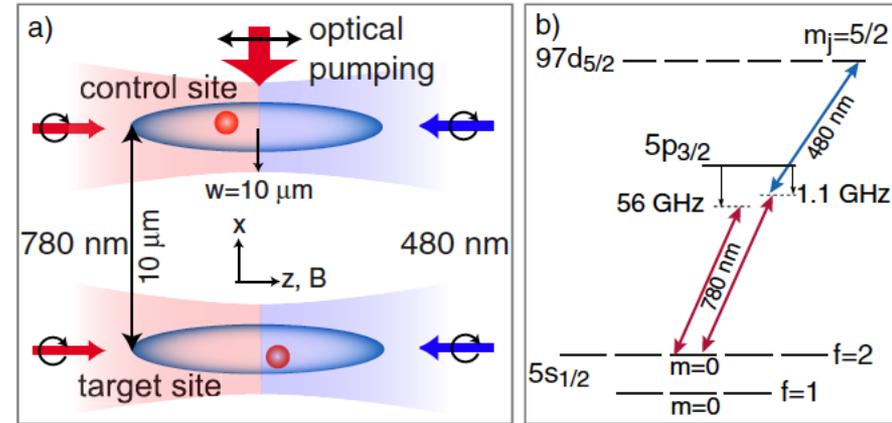
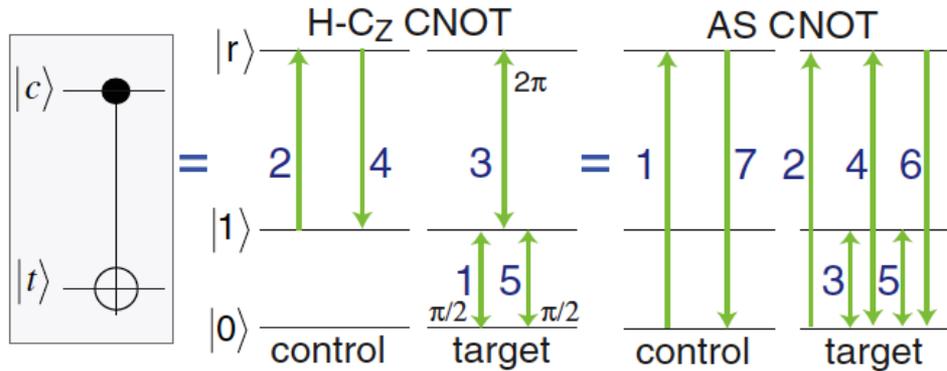
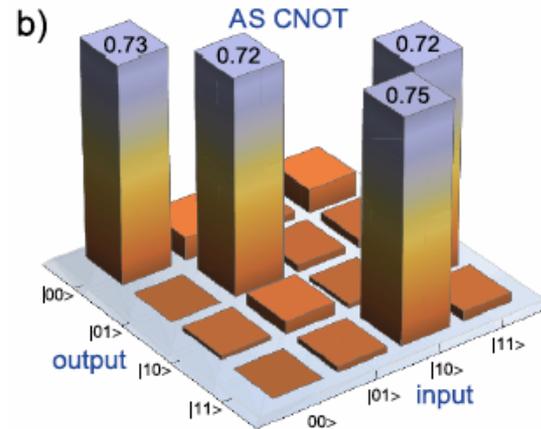
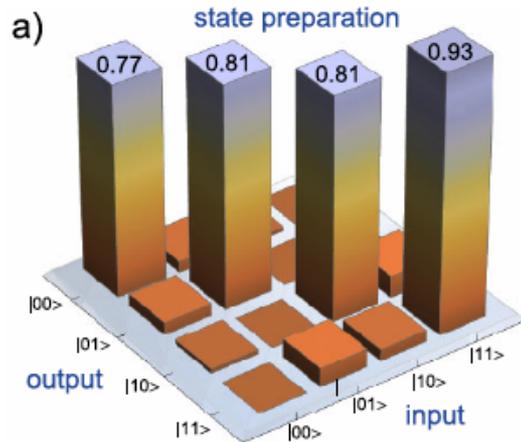


Table of truth: check blockade



$F = 0.73$

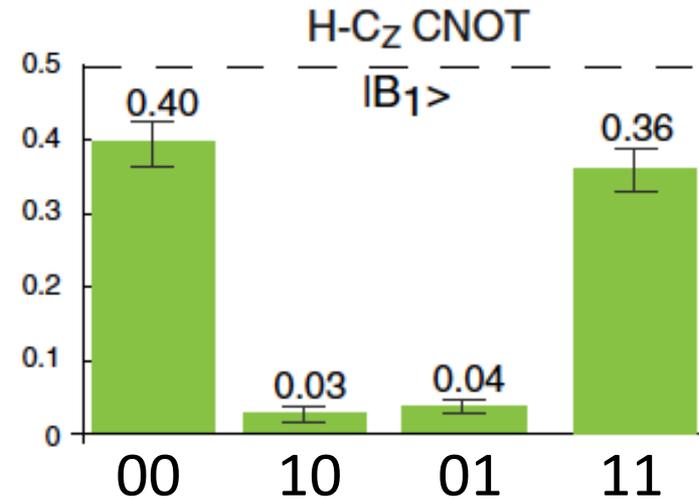
The CNOT gate at uni. Wisconsin (2)

Prepare entangled states

Prepare $(|0\rangle + |1\rangle)|0\rangle$

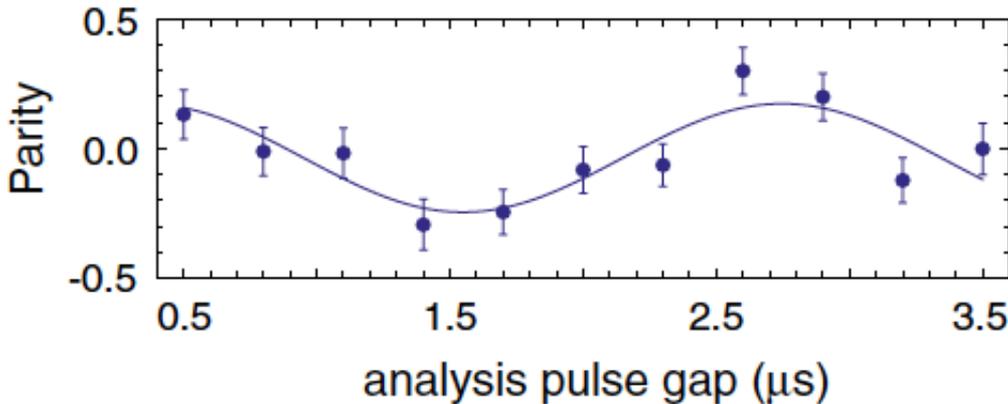


gate



Check coherence: global Raman rotation

$$\Pi = P_{00} + P_{11} - P_{01} - P_{10}$$



Amplitude

$$\Rightarrow \Re(\rho_{0011})$$

$$\mathcal{F} = \frac{1}{2}(P_{00} + P_{11}) + \Re(\rho_{0011})$$

$$\mathcal{F} = 0.58$$

(with loss correction)

Tailoring the interaction: the dressed interaction picture

PHYSICAL REVIEW A, VOLUME 65, 041803(R)

Spin squeezing of atoms by the dipole interaction in virtually excited Rydberg states

Isabelle Bouchoule and Klaus Mølmer

PRL **104**, 223002 (2010)

PHYSICAL REVIEW LETTERS

week ending
4 JUNE 2010

Strongly Correlated Gases of Rydberg-Dressed Atoms: Quantum and Classical Dynamics

G. Pupillo,¹ A. Micheli,¹ M. Boninsegni,^{2,1} I. Lesanovsky,³ and P. Zoller¹

PHYSICAL REVIEW A **82**, 033412 (2010)

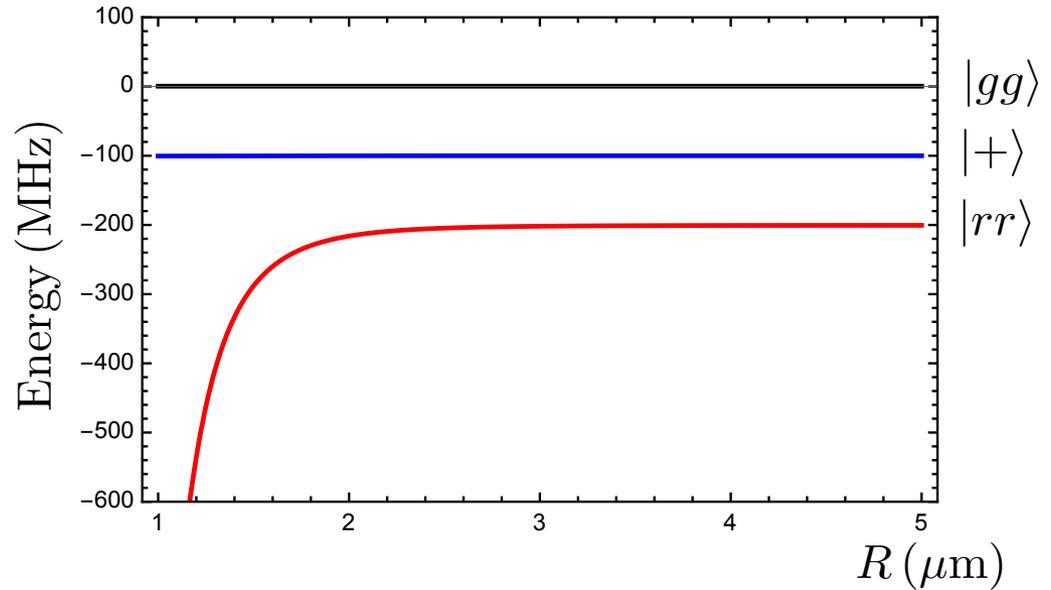
Interactions between Rydberg-dressed atoms

J. E. Johnson and S. L. Rolston

$$H = \hbar \begin{pmatrix} 0 & \frac{\Omega}{\sqrt{2}} & 0 \\ \frac{\Omega}{\sqrt{2}} & \delta & \frac{\Omega}{\sqrt{2}} \\ 0 & \frac{\Omega}{\sqrt{2}} & 2\delta + U_{dd} \end{pmatrix}.$$

$$U_{dd} = -\frac{1000(\text{MHz})}{R(\mu\text{m})^6}$$

$$\delta = -100 (\text{MHz}) ; \Omega = 10 (\text{MHz})$$



Tailoring the interaction: the dressed interaction picture

PHYSICAL REVIEW A, VOLUME 65, 041803(R)

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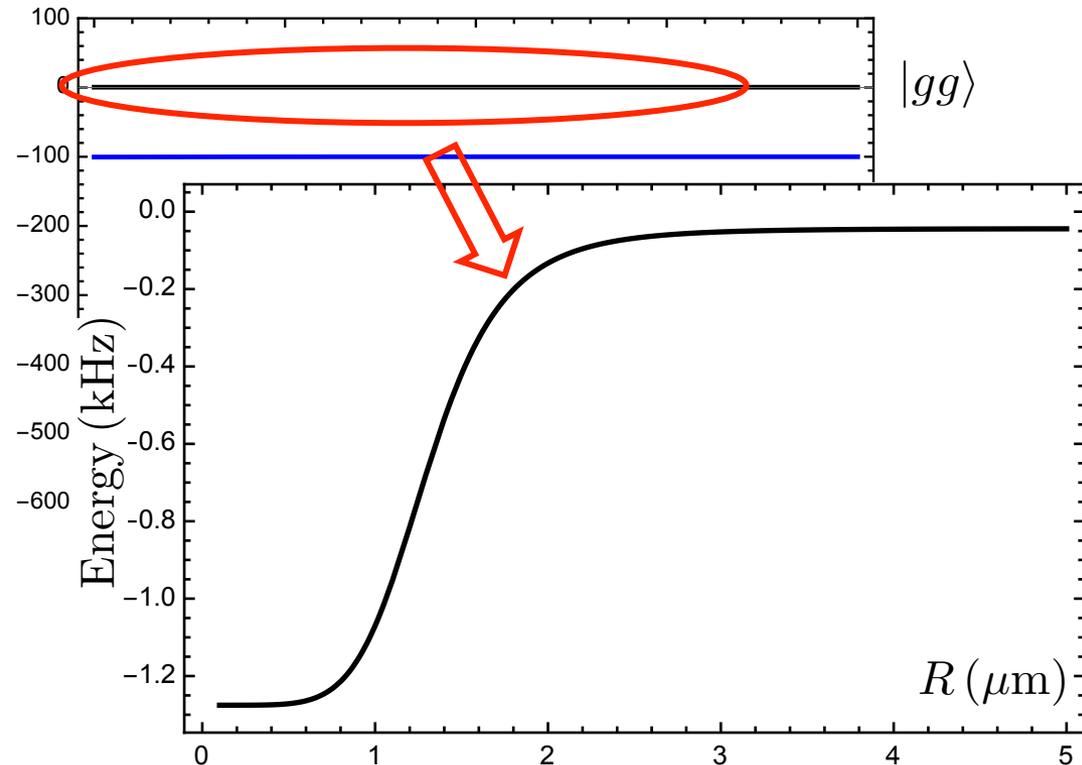
Interactions between Rydberg-dressed atoms

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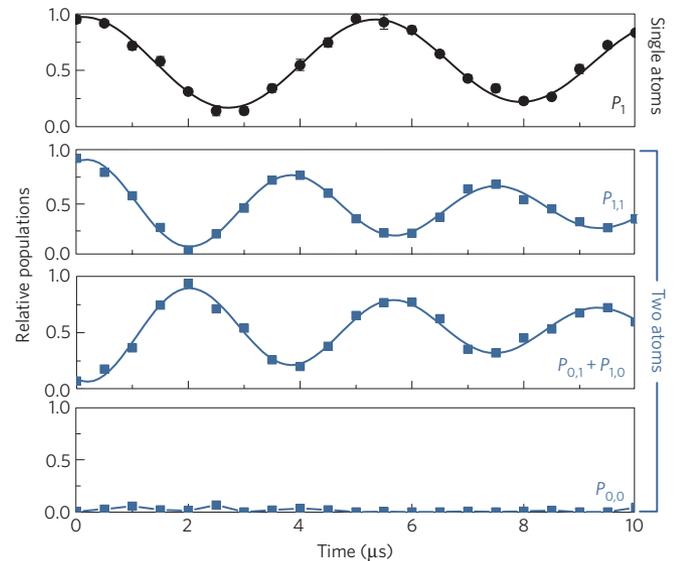
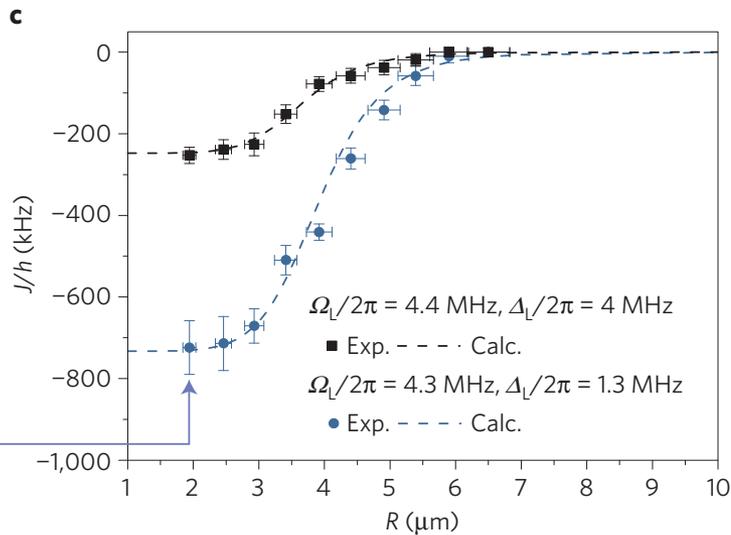
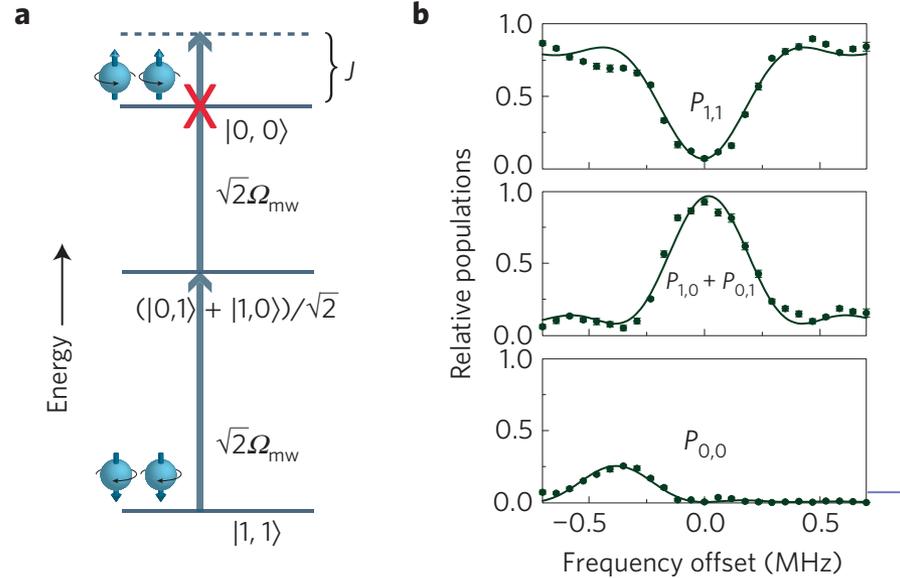
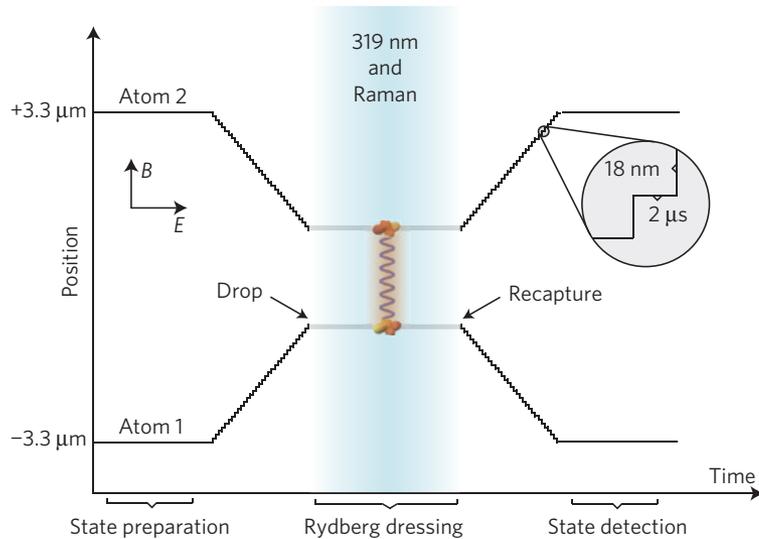
$$U_{dd} = -\frac{1000(\text{MHz})}{R(\mu\text{m})^6}$$

$$\delta = -100 (\text{MHz}) ; \Omega = 10 (\text{MHz})$$



Tailoring the interaction: Rydberg “dressing” with two atoms

Jau *et al.*, Nat. Phys. **12**, 71 (2015)

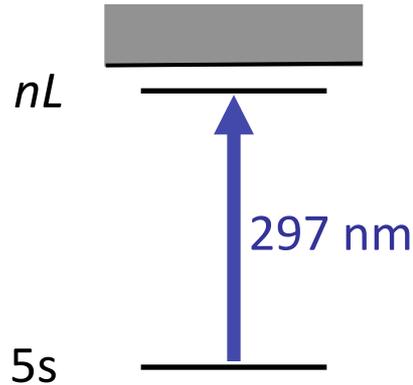


Blockade in atomic ensembles

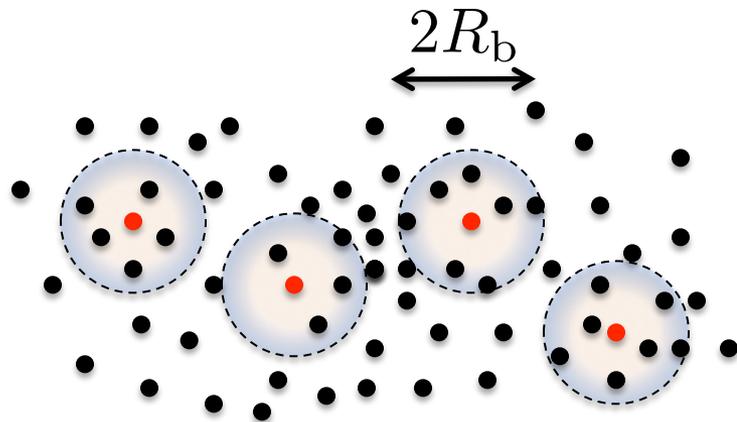
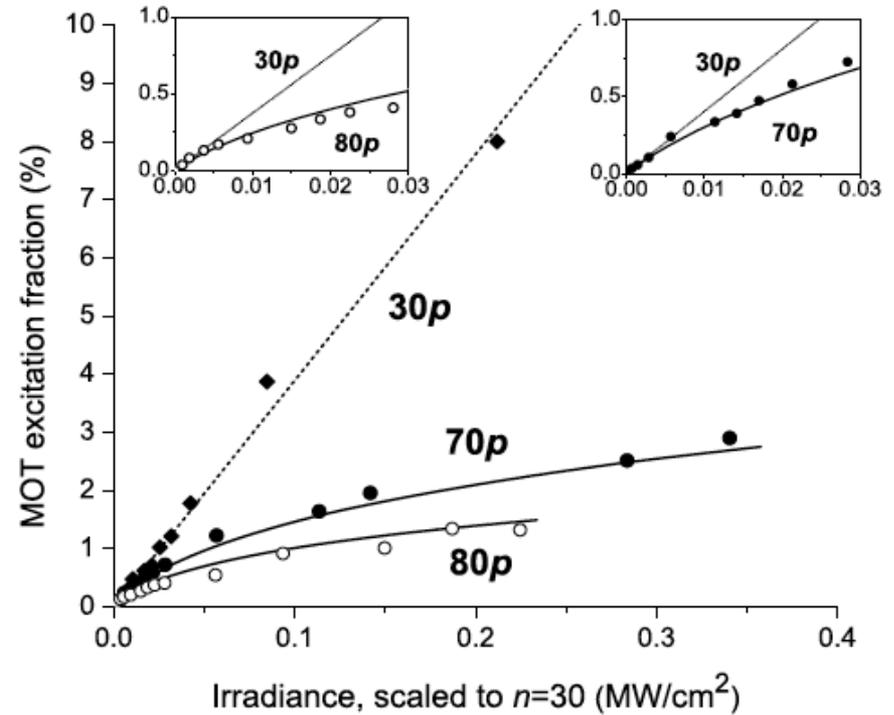
Applications to single-photon and
single-atom source

Rydberg blockade in cold atomic cloud: the U. Connecticut exp^t.

D. Tong *et al.*, PRL **93**, 063001 (2004)



Pulsed, incoherent laser excitation of a MOT
 \Rightarrow expect $N_{\text{Ryd}} \propto \text{Intensity}$

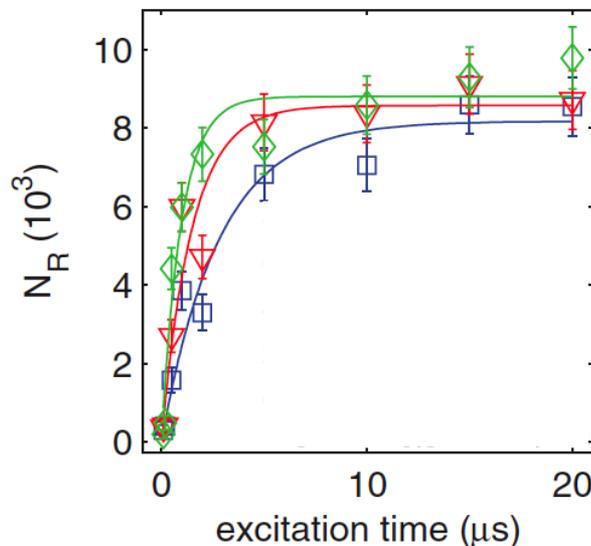


Increase $n \Rightarrow$ increase C_6
 \Rightarrow increase R_b

$$N_{\text{Ryd}}^{\text{max}} \approx \frac{\text{Volume}}{\frac{4\pi R_b^3}{3}}$$

Rydberg blockade in dense cold atomic cloud: the Stuttgart exp^t.

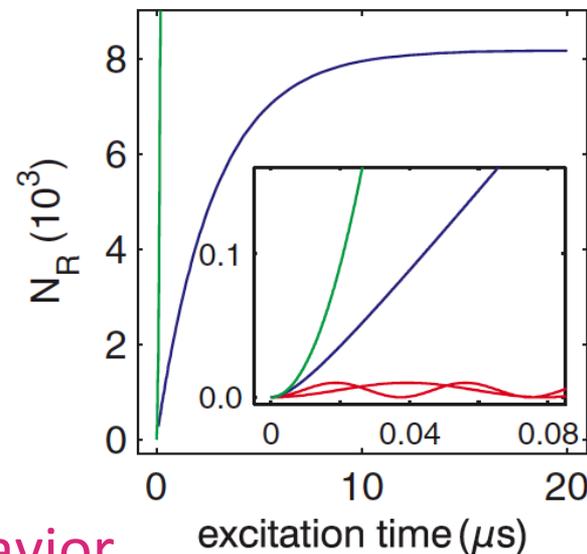
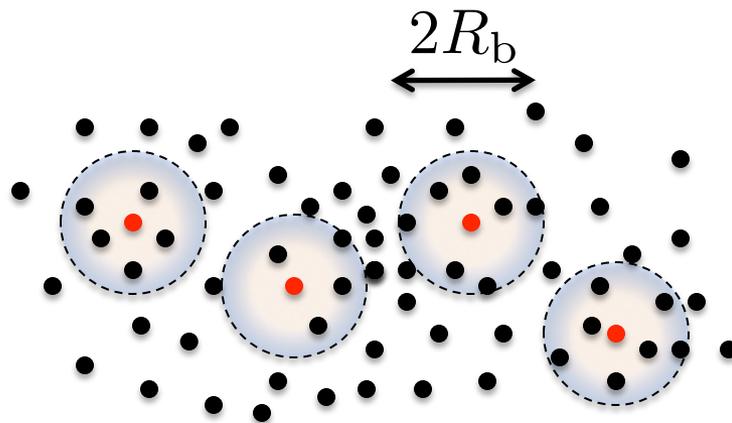
Use a dense ultracold cloud of ⁸⁷Rb + coherent 2-ph. excitation



⇒ reach saturation

H. Heidemann *et al.*,
PRL **99**, 163601 (2007)

Inhomogeneous
distribution of N



$$N_{\text{Ryd}}(t) = \sum_{\{N\}} C_N \sin^2 \frac{\Omega_0 \sqrt{N} t}{2}$$

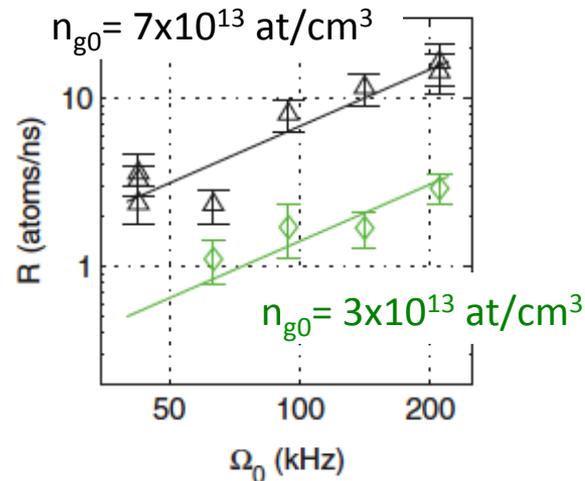
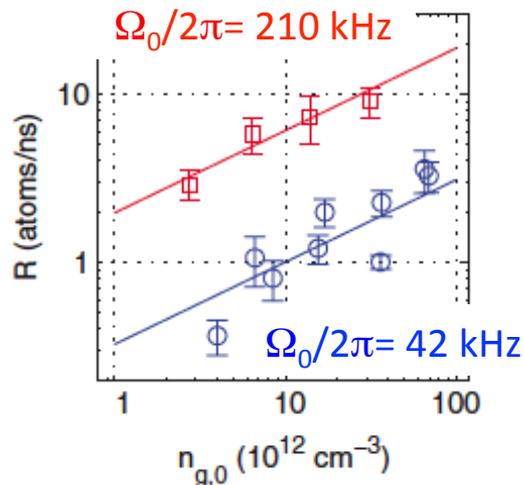
⇒ Incoherent behavior

Rydberg blockade in dense cold atomic cloud: the Stuttgart exp^t.

Check scaling laws

Expect rate of Rydberg production $R \propto \Omega_0 \sqrt{\langle N \rangle}$ with $\langle N \rangle \propto n_{g0}$

Find:

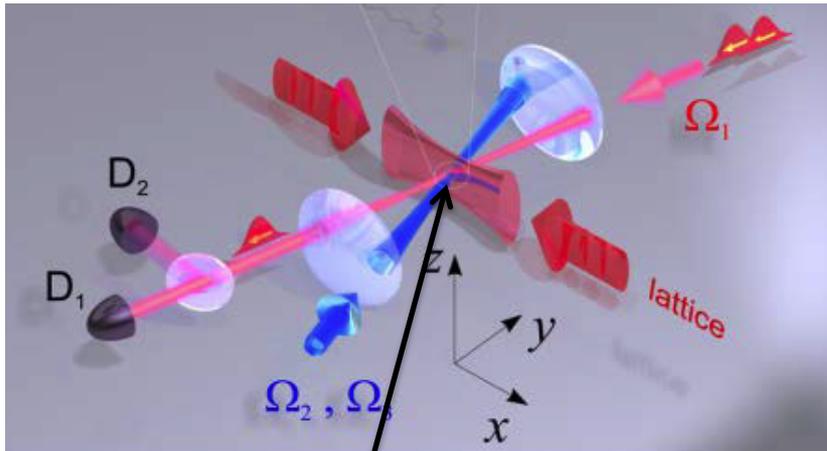


$$R \propto n_{g0}^{0.49} \Omega^{1.1}$$

Also, expect: $N_{\text{Ryd}}^{\text{max}} \propto \frac{1}{R_b^3} \propto \Omega_0^{1/2}$ and find: $\Omega_0^{0.38}$

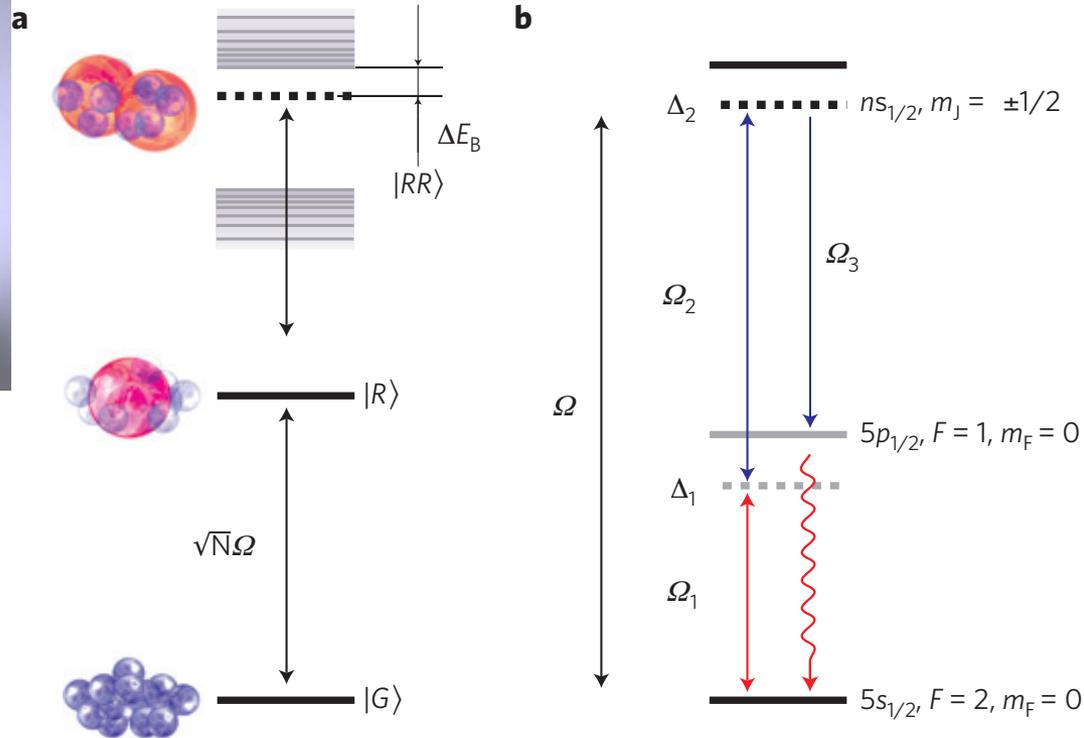
Collective Rabi oscillations in ensemble

Y.O. Dudin *et al.*, Nat. Phys. **8**, 790 (2012)



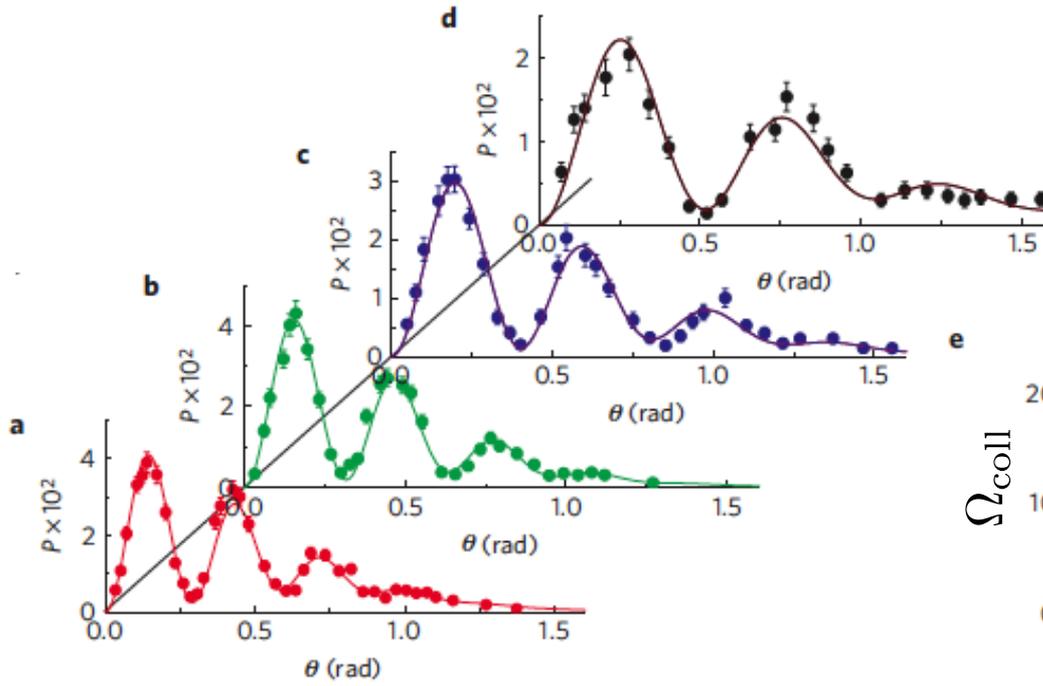
Excitation volume $< R_b^3$

102s, $R_b = 15 \mu\text{m}$



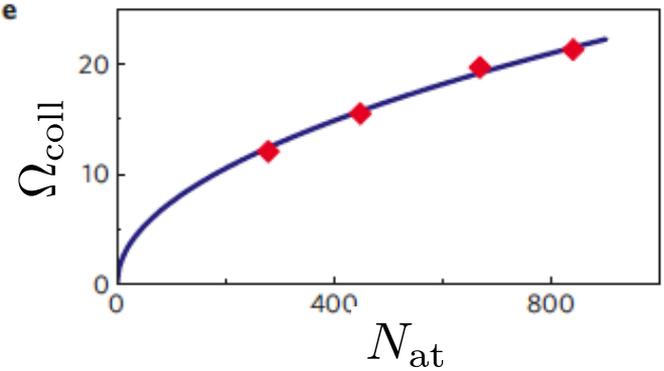
Collective Rabi oscillations in ensemble

Y.O. Dudin *et al.*, Nat. Phys. **8**, 790 (2012)



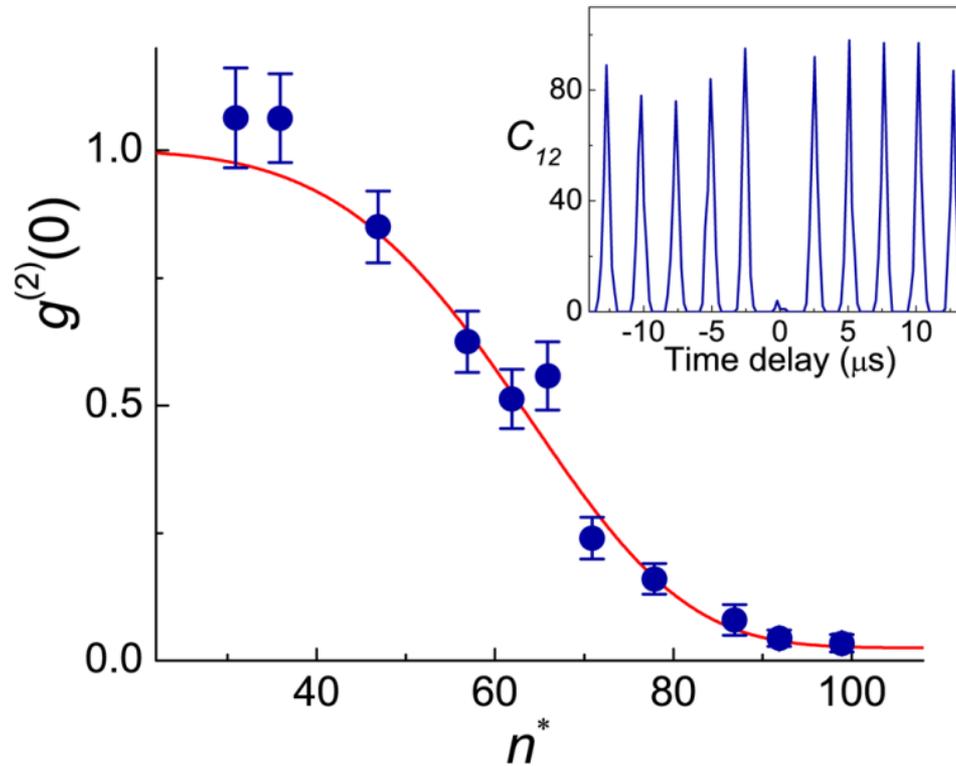
Expect:

$$\Omega_{\text{coll}} \propto \sqrt{N_{\text{at}}}$$

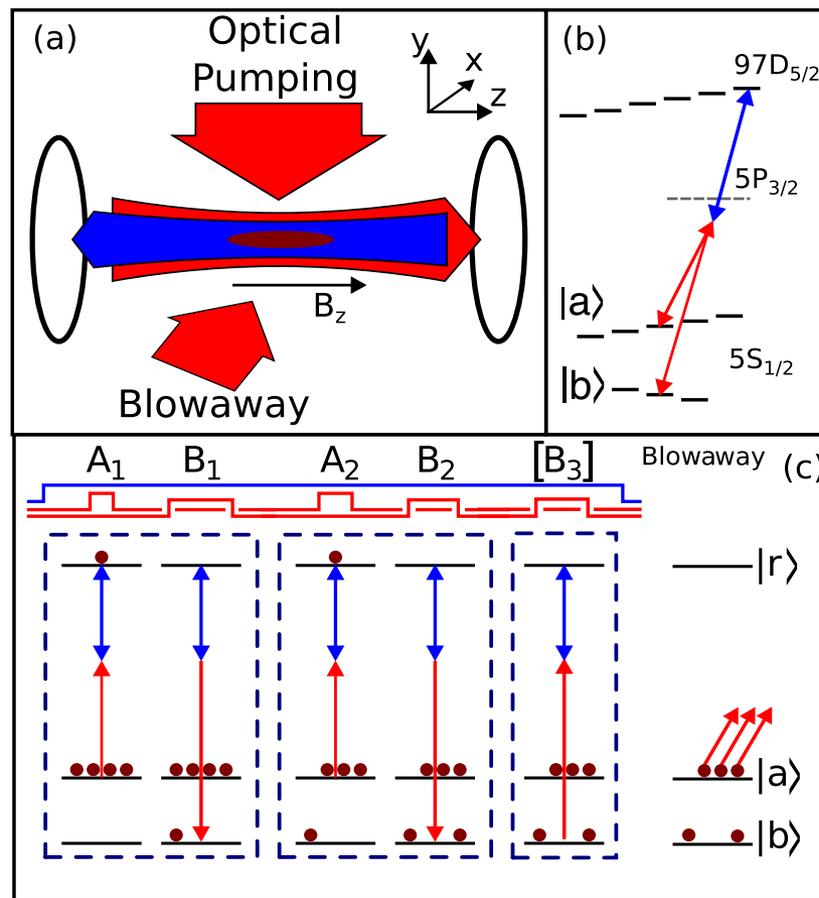
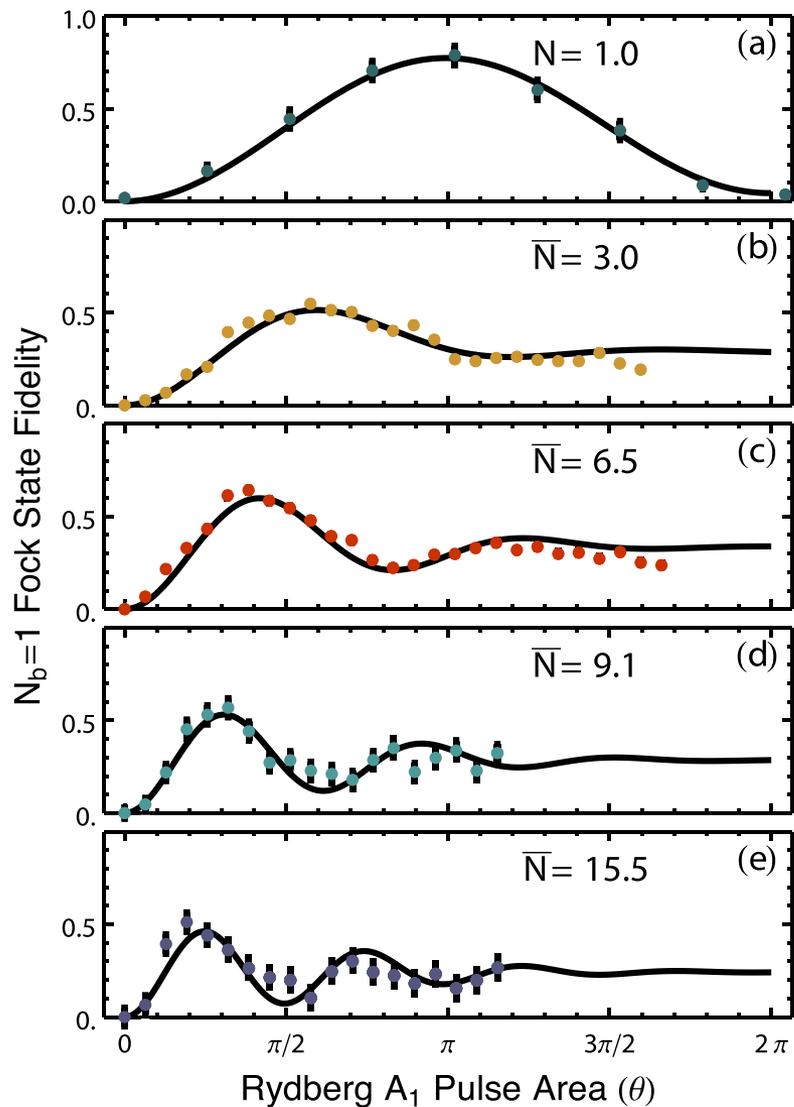


Application of blockade: a single photon source

Y.O. Dudin *et al.*, Science **336**, 887 (2012)



Atom Fock state preparation using blockade



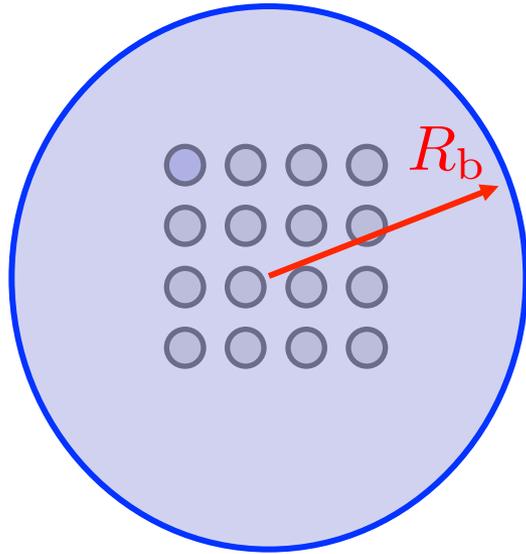
Outline

Lecture 1: Rydberg atoms and their interaction

Lecture 2: Rydberg Blockade and application to QIP

Lecture 3: Quantum simulation & Quantum Optics with Rydbergs

From blockade to many-body physics

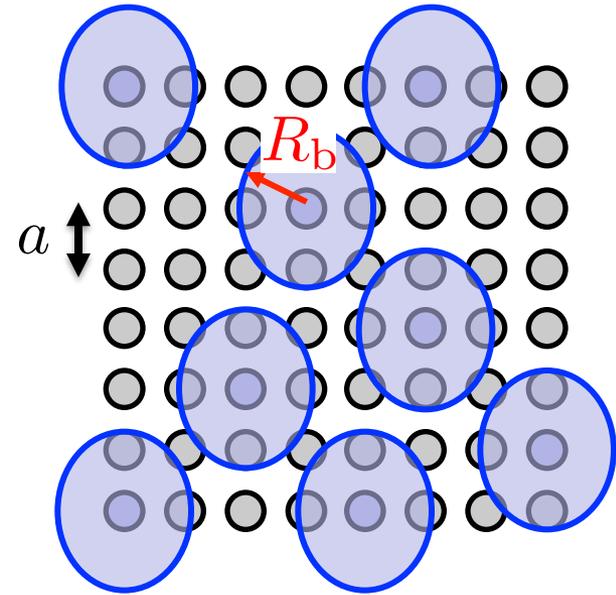


$\dim H = 2$

Two (collective) states

$$|ggg\dots\rangle \iff \frac{1}{\sqrt{N}} \sum_i |g\dots r_i g\dots\rangle$$

Also: Saffman, Kuzmich, Bloch, Pfau, Ott...



$\dim H \sim 2^N$

**Strongly correlated
many-body system!**

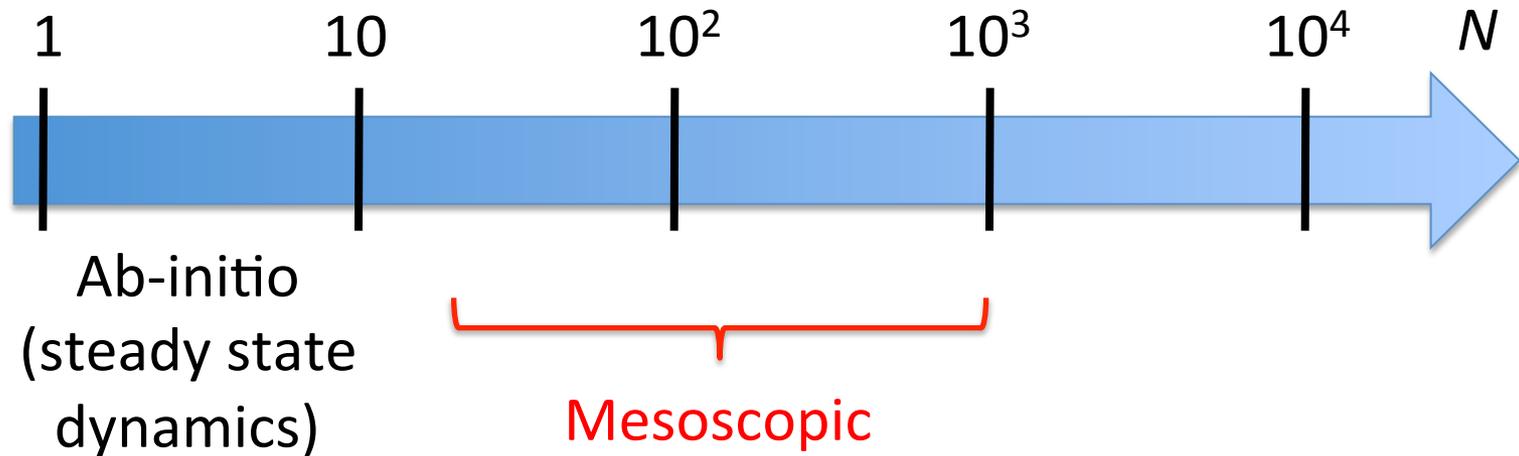
Experimentally :

$$\text{tune } \frac{R_b}{a} = 1-20$$

Many - body systems and complexity

Complexity: for $N > 30 - 40$, ab-initio calculations impossible!!

Ex: spin $1/2 \Rightarrow$ size of Hilbert space $\sim 2^N$ too large



Approximate methods ($10^2 < N < 10^5$): DMRG, Monte Carlo, density functionnal, mean field...

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

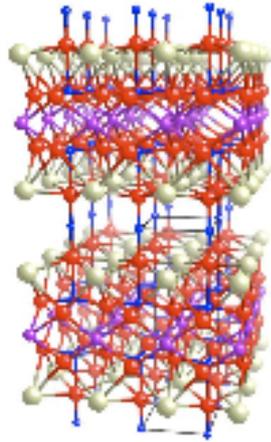
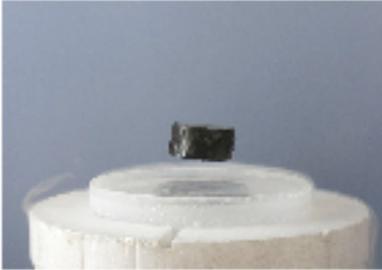
Simulating Physics with Computers

Richard P. Feynman

4. QUANTUM COMPUTERS—UNIVERSAL QUANTUM SIMULATORS

... (I mean each atom is just a point which has numbers associated with it, with quantum-mechanical rules). For example, the spin waves in a spin lattice imitating Bose-particles in the field theory. I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. But I don't know whether the general theory of this intersimulation of quantum systems has ever been worked out, and so I present that as another interesting problem: to work out the classes of different kinds of quantum mechanical systems which are really intersimulatable—which are equivalent—as has been done in the case of classical computers. It has been found that there is a kind of

Quantum simulation: an example



Ex: high- T_c superconductivity

Experience
on the real
system

simplify



Model Hamiltonian

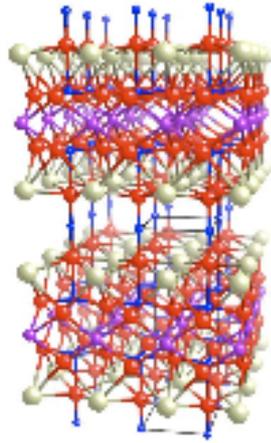
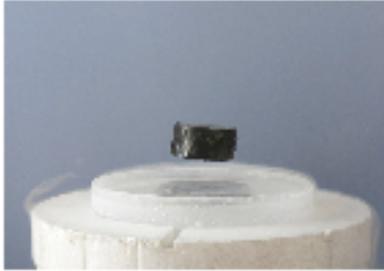
$$H_{\text{model}} = - \sum_{i,j} J_{ij} a_i^\dagger a_j + \sum_i g (a_i^\dagger)^2 (a_i)^2$$


Calculating H_{model}
 \Rightarrow superconductivity?



Calculation too hard...

Quantum simulation: an example



Ex: high- T_c superconductivity

**Experience
on the real
system**

simplify

Model Hamiltonian

$$H_{\text{model}} = - \sum_{i,j} J_{ij} a_i^\dagger a_j + \sum_i g (a_i^\dagger)^2 (a_i)^2$$

Lab...

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar} H_{\text{model}} t} |\psi(0)\rangle$$

Mesure outcome
of simulator:
**ground state =
supercond.?**

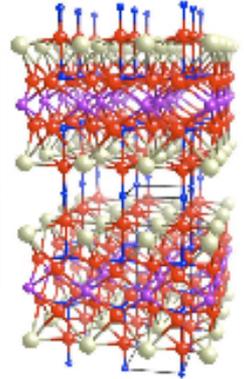
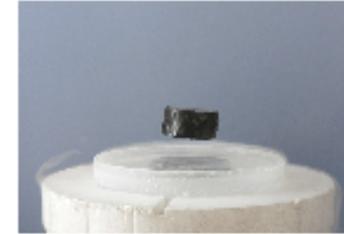
“Preparable”

**Quantum simulator =
engineer ensemble of
atoms ruled by H_{model}**

What can you simulate = N-body problem?

High-Tc superconductivity

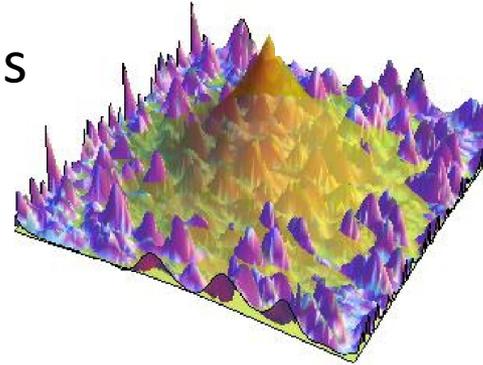
$$\text{Fermi-Hubbard: } \hat{H} = -t \sum_{\sigma, \langle i, j \rangle \in \Omega} (\hat{c}_{i, \sigma}^\dagger \hat{c}_{j, \sigma} + h.c.) + U \sum_{i \in \Omega} \hat{n}_{i, \downarrow} \hat{n}_{i, \uparrow},$$



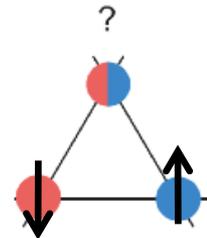
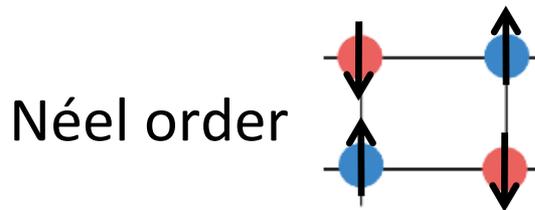
Conduction properties of metal:

Influence of disorder and interactions

Anderson
localization



Quantum magnetism: ferro & anti-ferromagnetic order

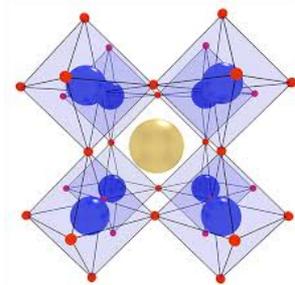
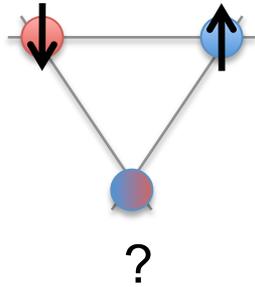


frustration Phase diagram,
dynamics...

+ Dirac equation, cosmology, gauge theory, quantum chemistry...

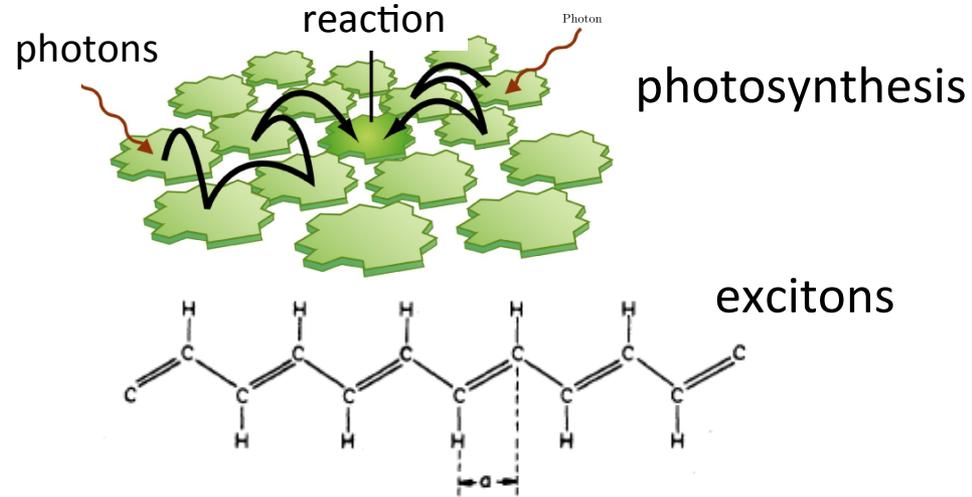
Spin models in condensed matter systems: a few examples

Quantum magnetism

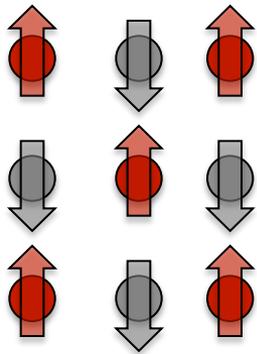


Perovskite
 $Y_2Ti_2O_7$

Transport of excitations



“Simplest” system: interacting spin $\frac{1}{2}$ particles on a lattice:



Ising

$$\hat{H} = \sum_{i \neq j} J_{ij} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}$$

XY model

$$\hat{H} = \sum_{i \neq j} J_{ij} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$$

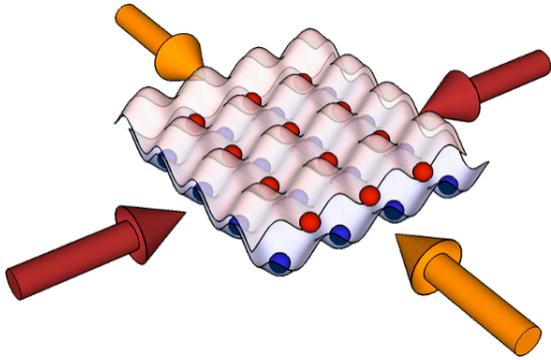
Open questions (long-range interaction) for $N > 30$:

phase diagram, dynamics, role anisotropy, geometry (frustration) ...

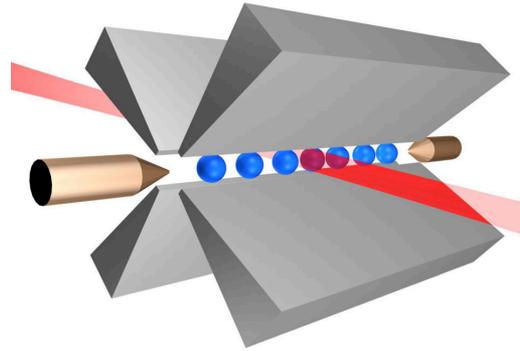
Quantum state engineering

Current status

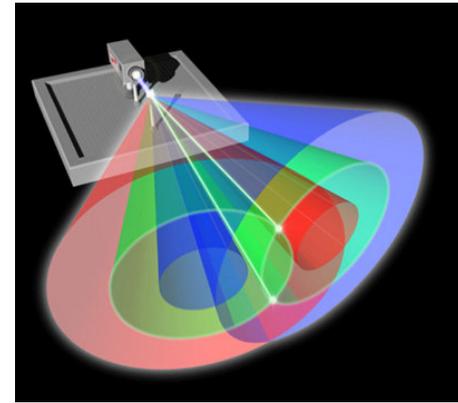
- ✓ Isolate and control ≤ 10 individual quantum systems



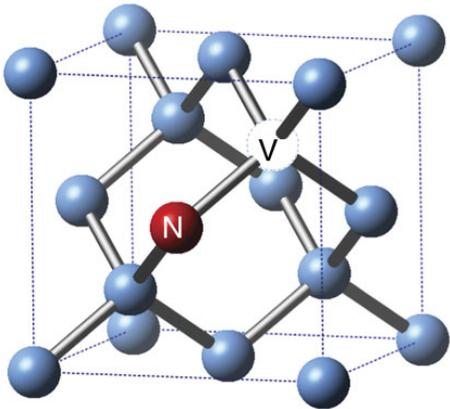
Neutral atoms



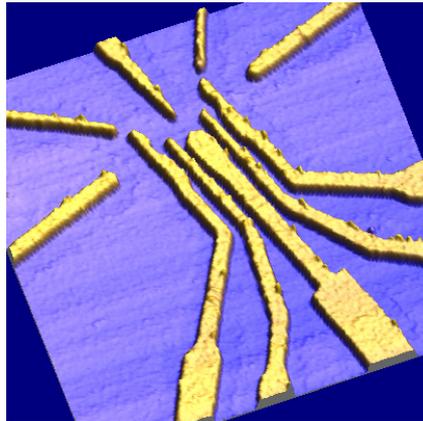
Trapped ions



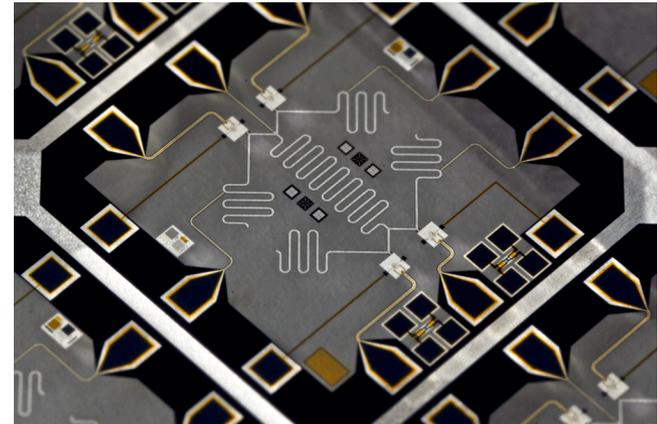
Photons



NV centers



Quantum dots



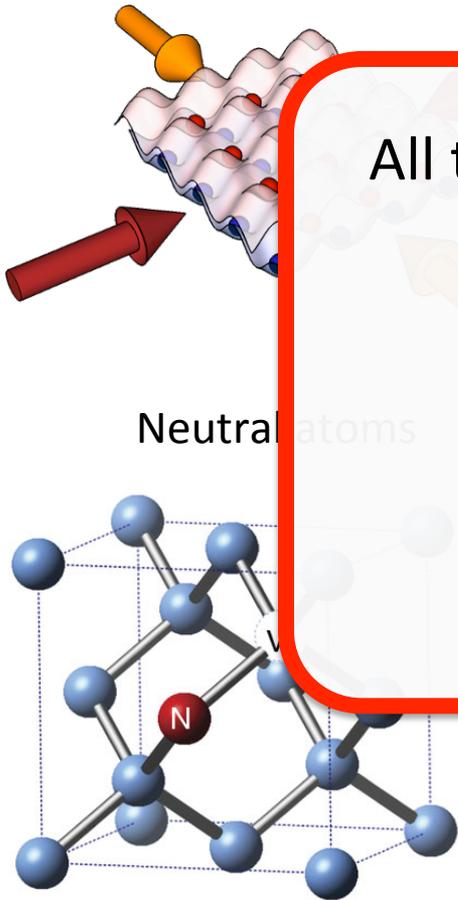
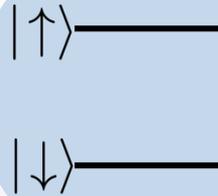
Superconducting qubits

Quantum state engineering

Current status

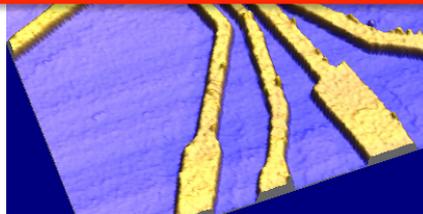
- ✓ Isolate and control ≤ 10 individual quantum systems

All those systems can be used as two-level systems to encode a spin:

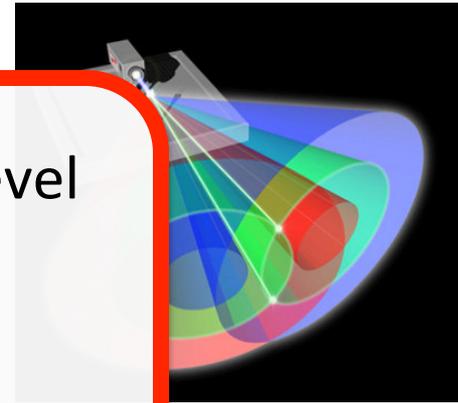


Neutral atoms

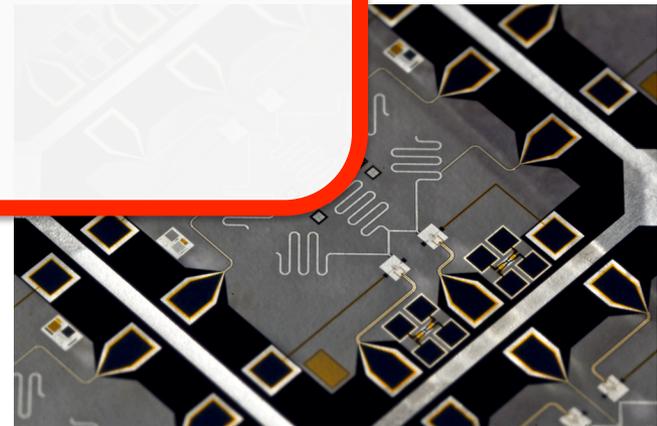
NV centers



Quantum dots

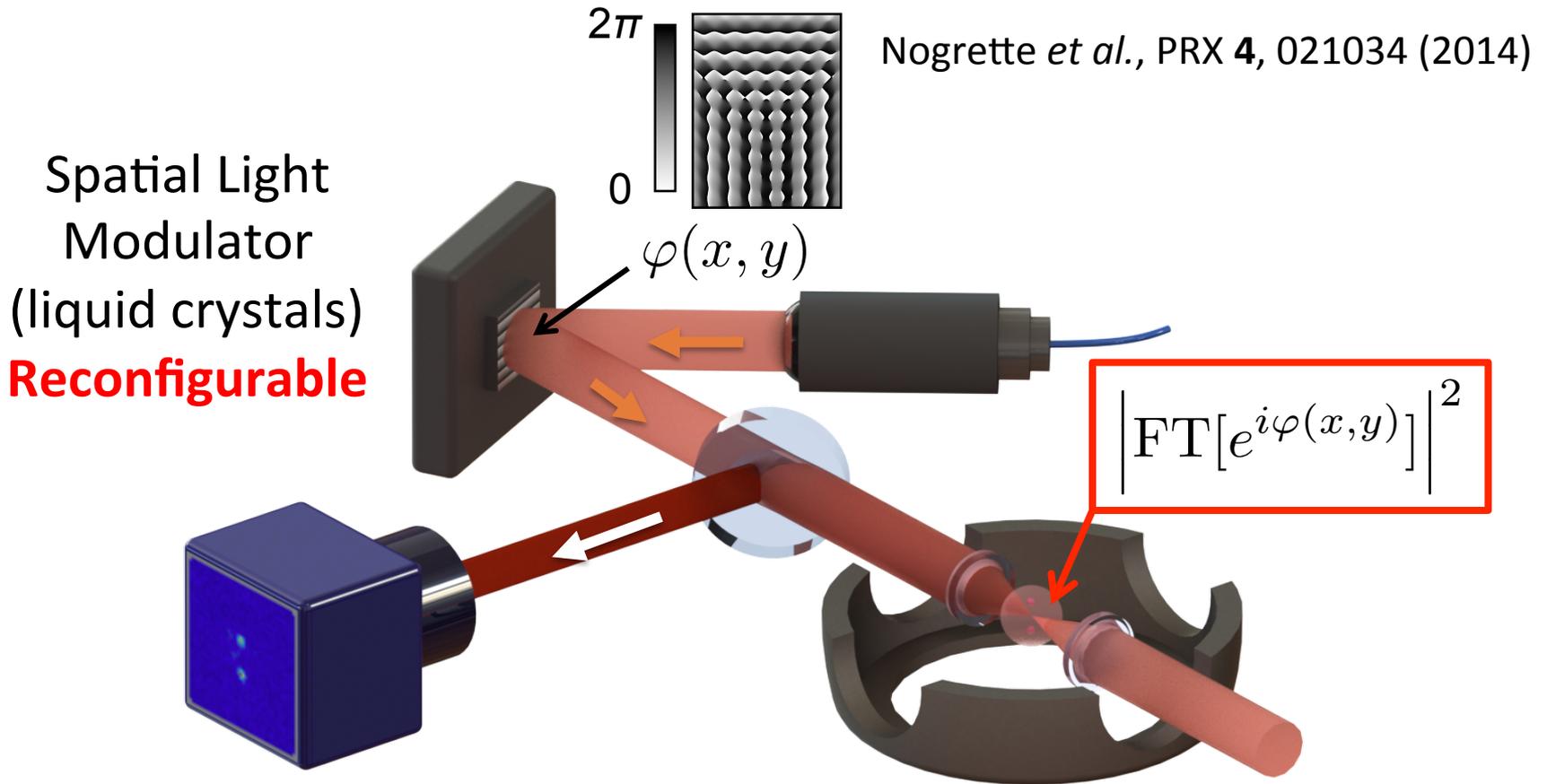


Photons



Superconducting qubits

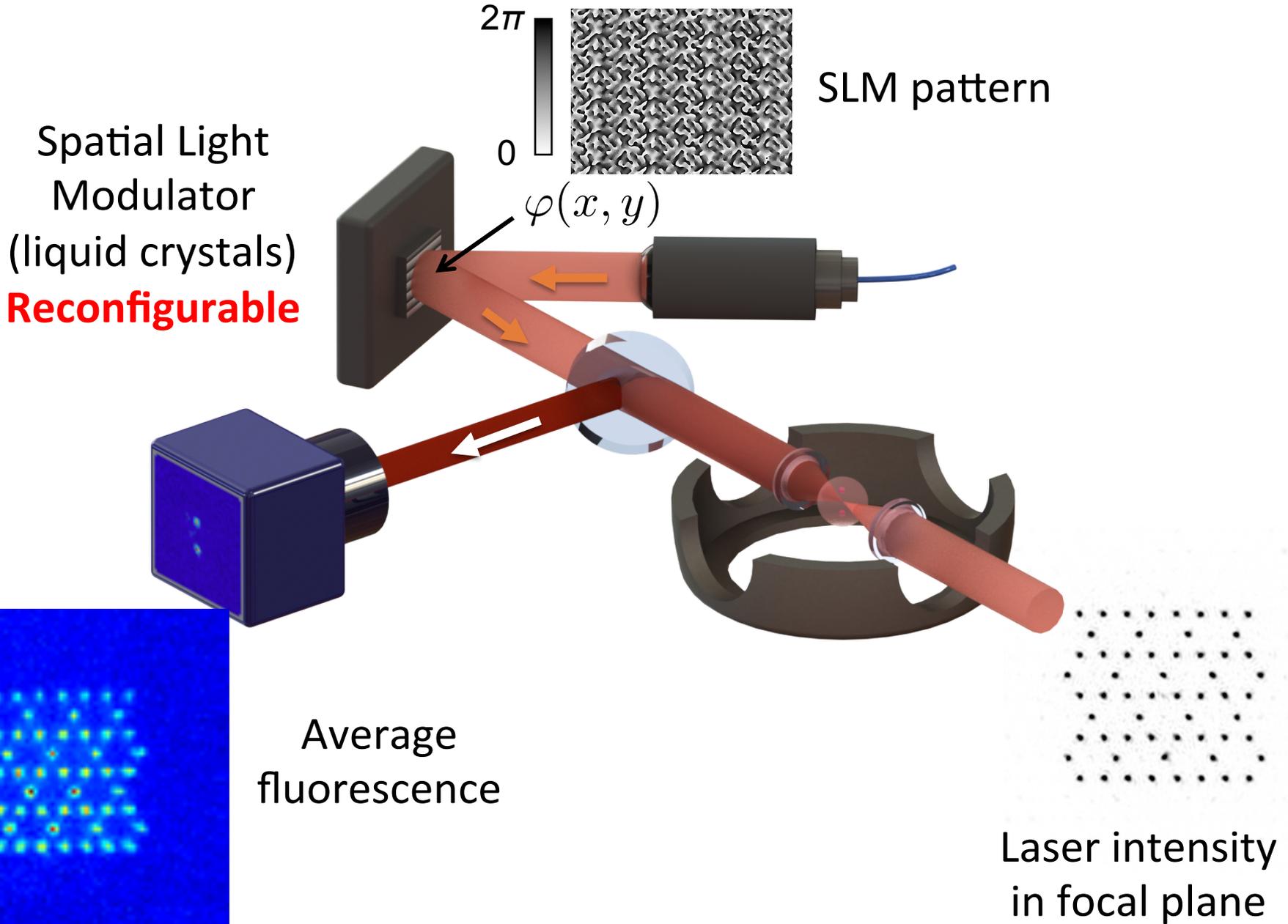
Holographic 2D arrays of tweezers



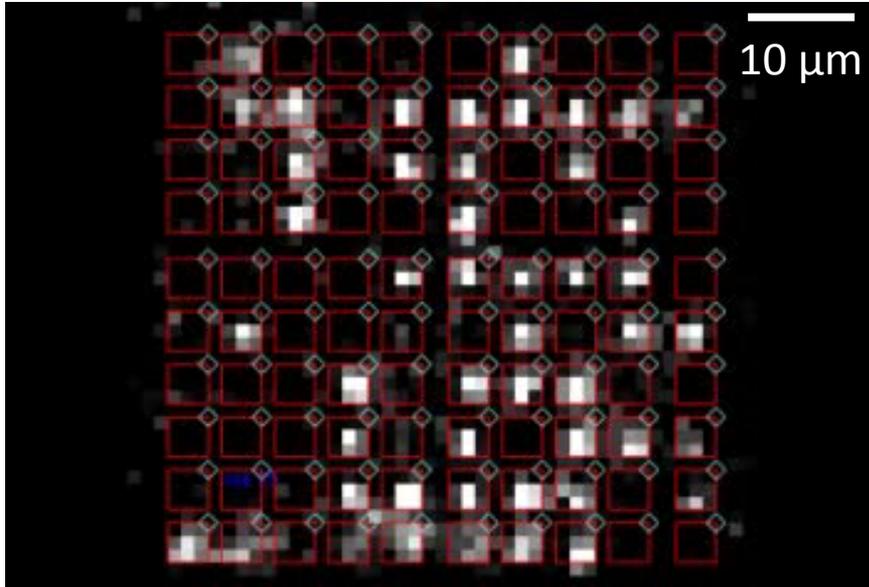
Phase calculation: iterative algorithm [Gerchberg – Saxton, Optik **35**, 237 (1972)]

Related works: Darmstadt, Amsterdam, Wisconsin, Harvard, Albuquerque, Chofu, Otago...

Holographic 2D arrays of tweezers



Arrays of optical tweezers with individual atoms

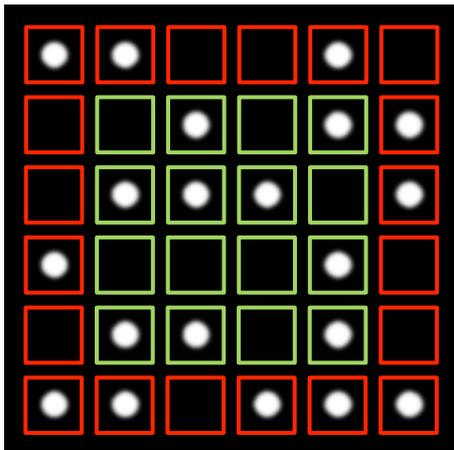


Problem: stochastic loading ($p \sim 0.5$)

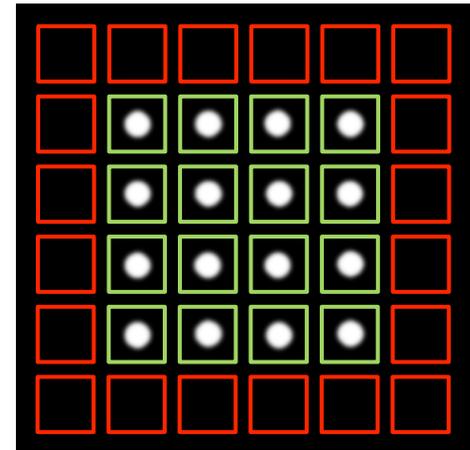
One solution: **sort atoms in arrays**

Miroshnychenko, Nature **442**, 151 (2006)

Initial atom distribution
(stochastically filled)

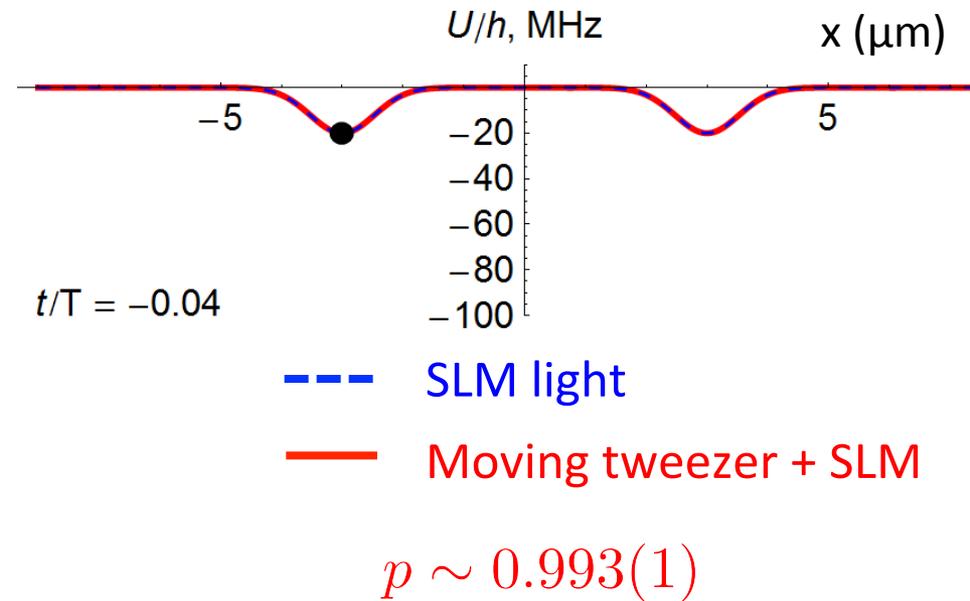
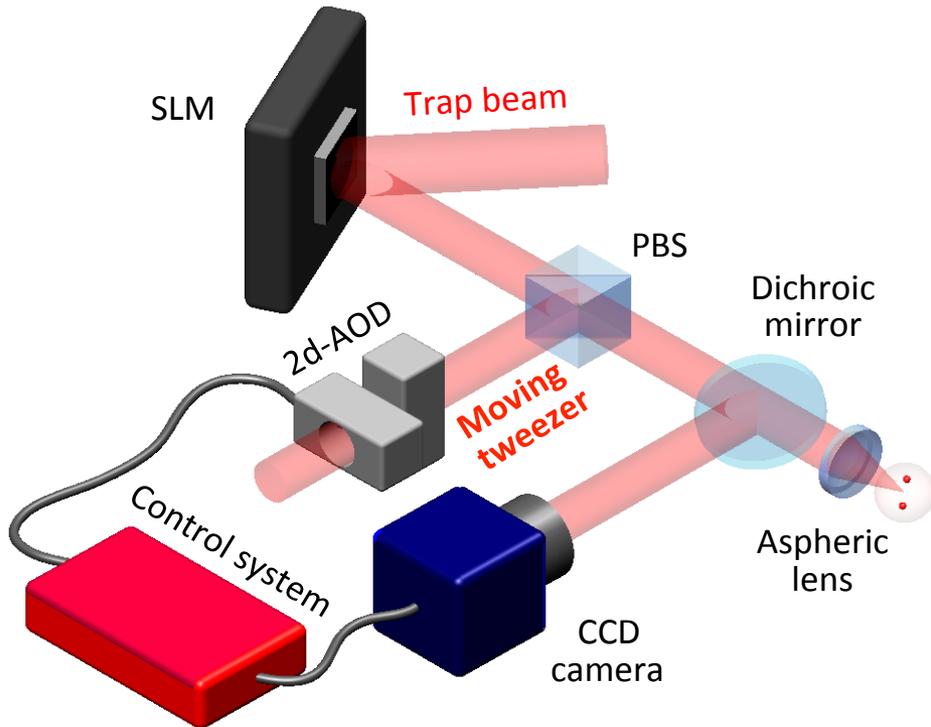


Target atom distribution
(ordered array)



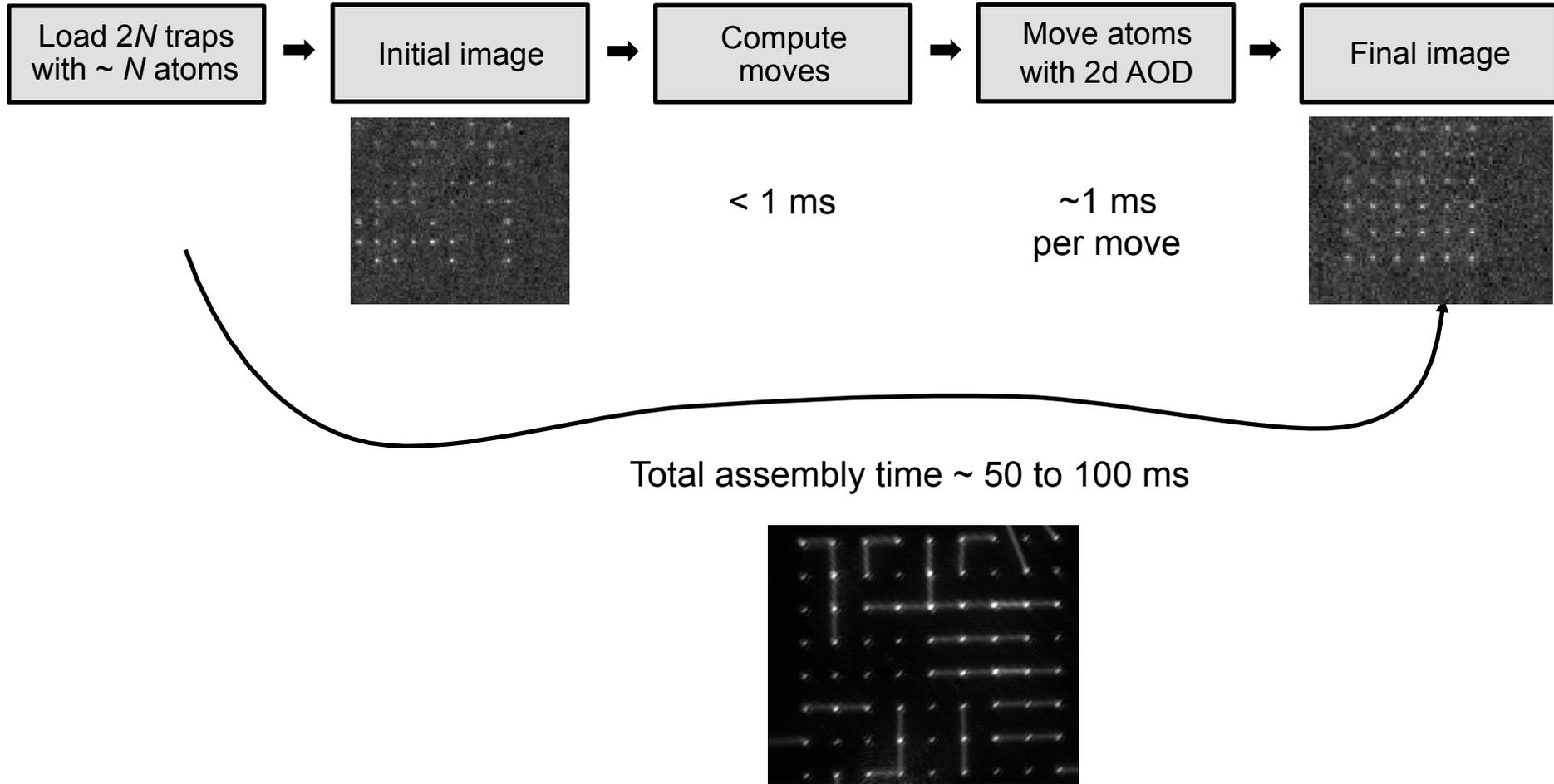
Moving optical tweezers for atom assembling

Barredo, de Léséleuc, *et al.*, Science **354**, 1021 (2016)



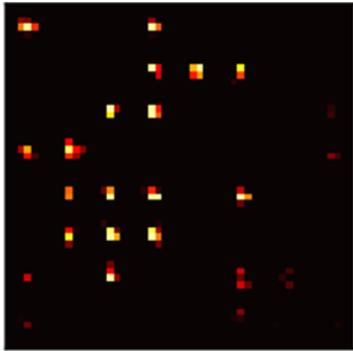
Labuhn, PRA **90**, 023415 (2014)

Atom assembler sequence

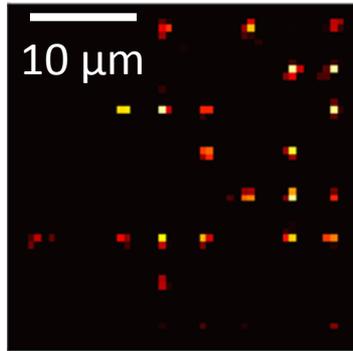


Gallery of assembled 2D arrays... (single-shot images...)

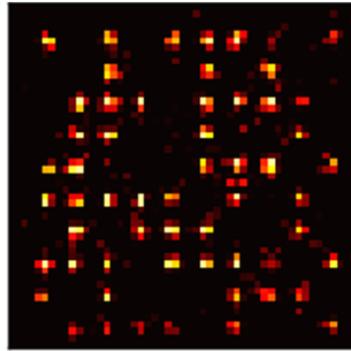
Initial



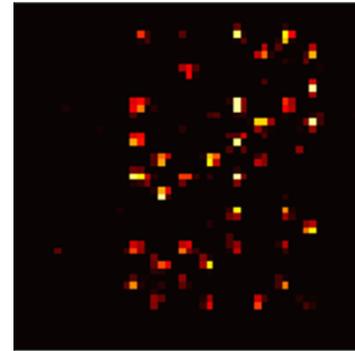
14 moves



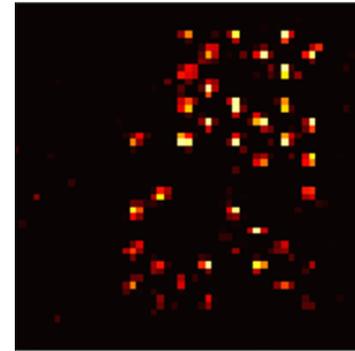
15 moves



53 moves

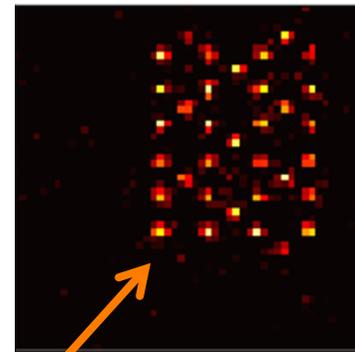
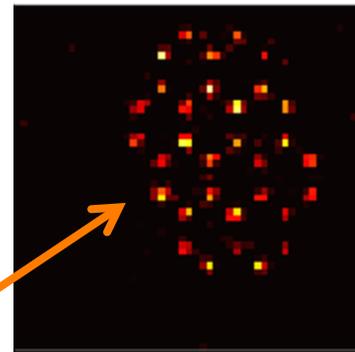
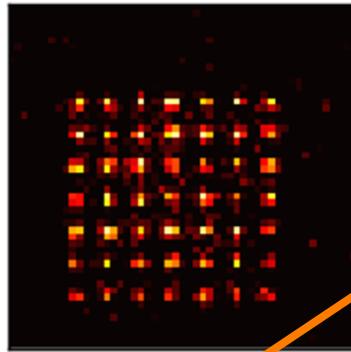
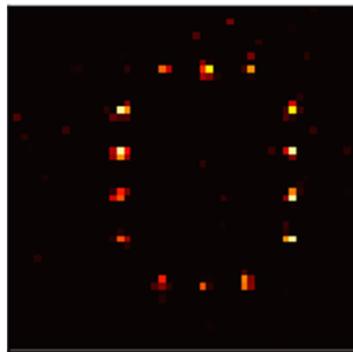
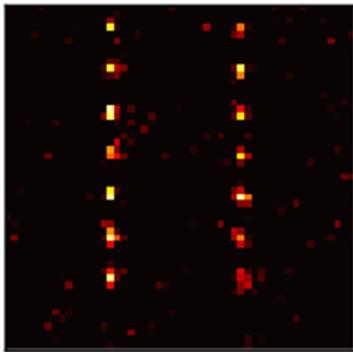


41 moves

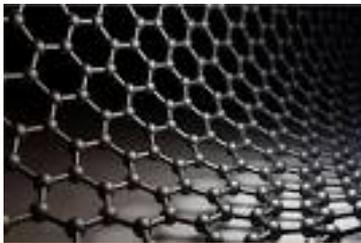


43 moves

Final

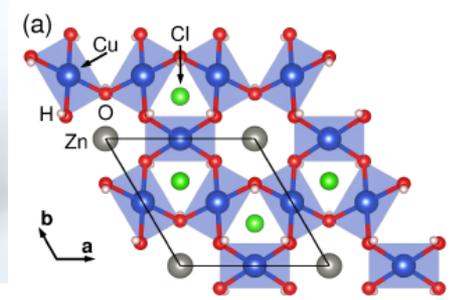


Hexagonal

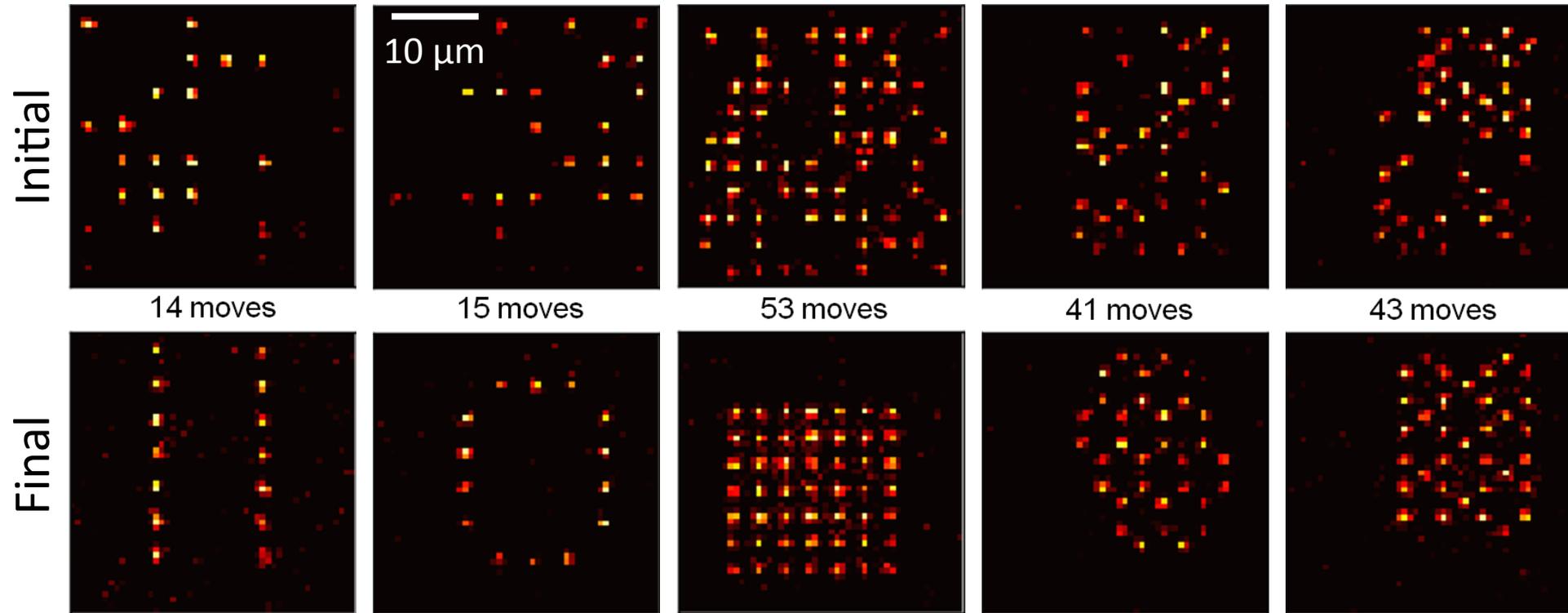


graphene

Kagome: Herbertsmithite



Gallery of assembled 2D arrays... (single-shot images...)



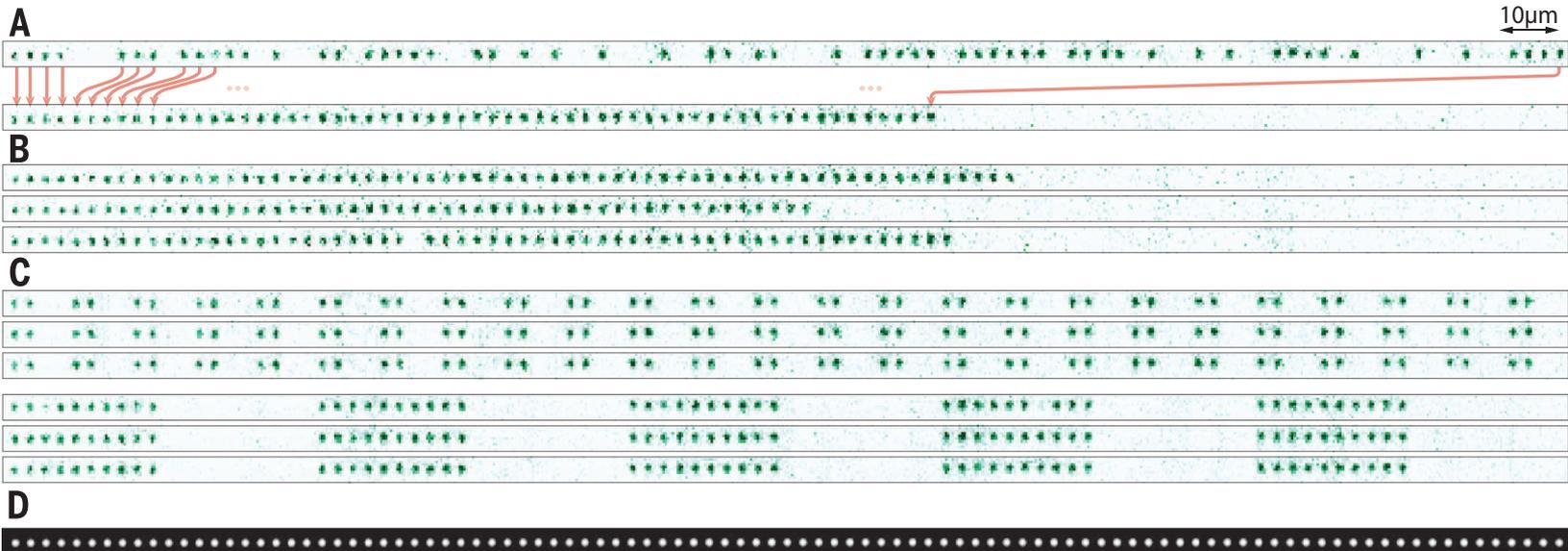
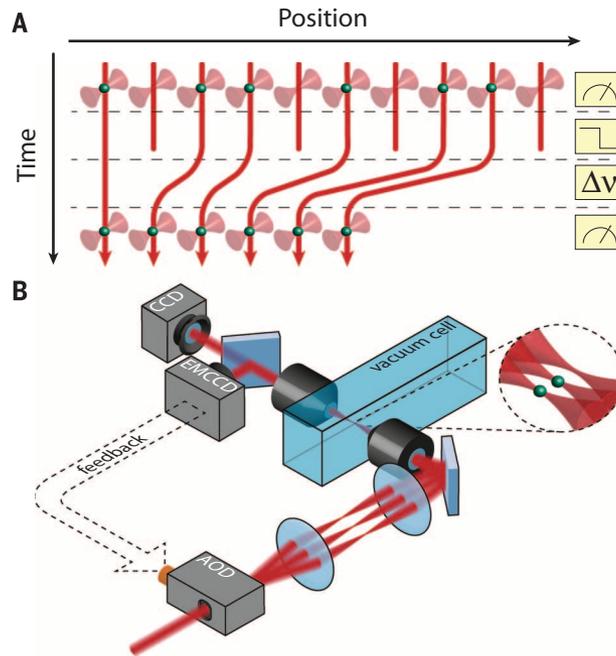
Barredo, de Léséleuc, *et al.*, *Science* **354**, 1021 (2016)

- Fully loaded arrays up to 50 atoms
- 98% filling fraction ~ 1 / sec rep. rate
- 100% filling every ~ 2 -5 sec

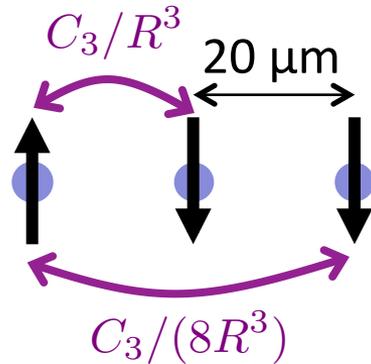
Related work in Harvard (1D), *Science* **354**, 1024 (2016)

Sorting in 1D (Harvard)

Science **354**,1024 (2016)

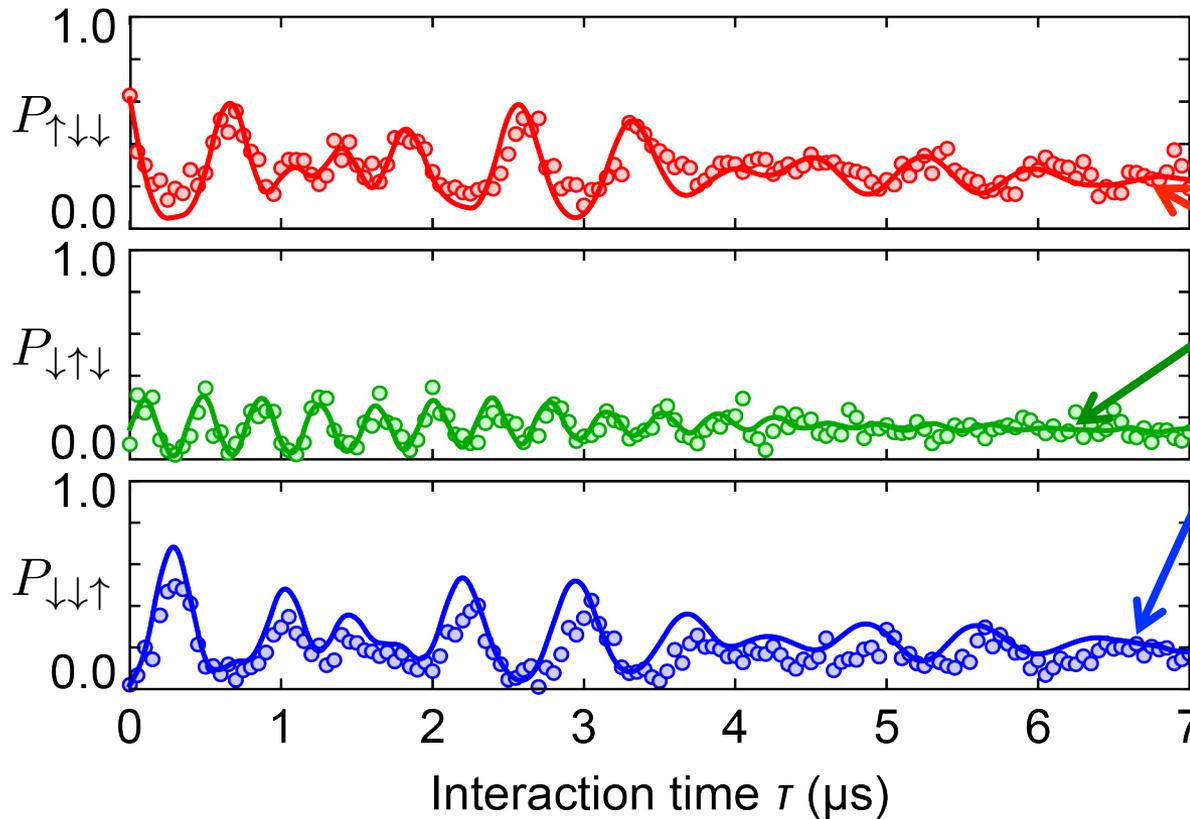


Observation of spin exchange in a 3-atom chain



D. Barredo *et al.*, PRL **114**, 113002 (2015)

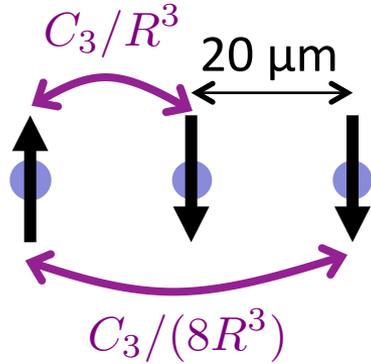
Prepare $|\uparrow\downarrow\downarrow\rangle$ at $t = 0$,
and let the system evolve



XY model
(Schrödinger)
+ imperfections

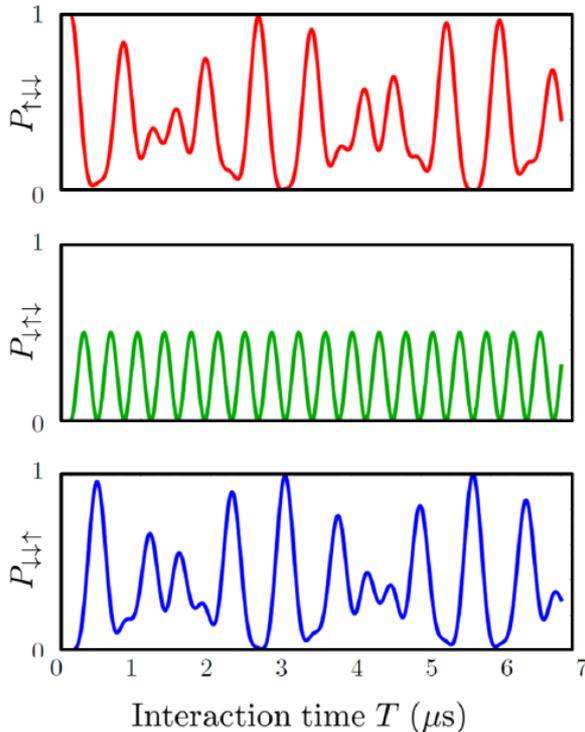
No adjustable
parameter

Three-atom “spin-chain”: what to expect (theory) ?



Prepare $|\uparrow\downarrow\downarrow\rangle$ at $t = 0$,
and let the system evolve

$1/R^3$ interaction



2 off-diagonal couplings: V & $V / 8$

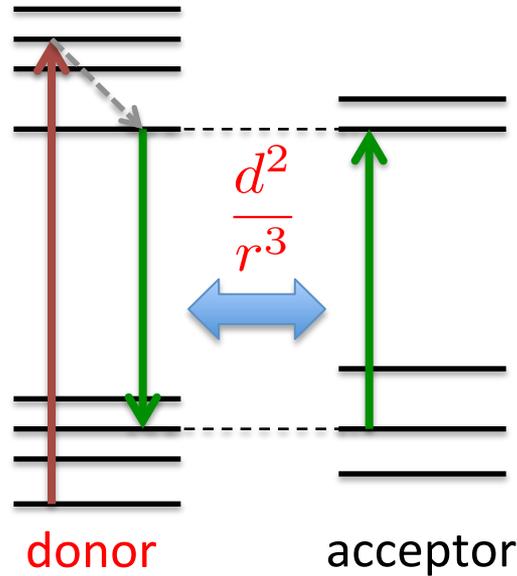
\Rightarrow eigenvalues (incommensurate):

$$\frac{V}{16} \left(1 + 3\sqrt{57} \right) , \quad \frac{V}{16} \left(1 - 3\sqrt{57} \right) , \quad -\frac{V}{8}$$

Resonant energy exchange around us...

Energy transport in biological systems

FRET



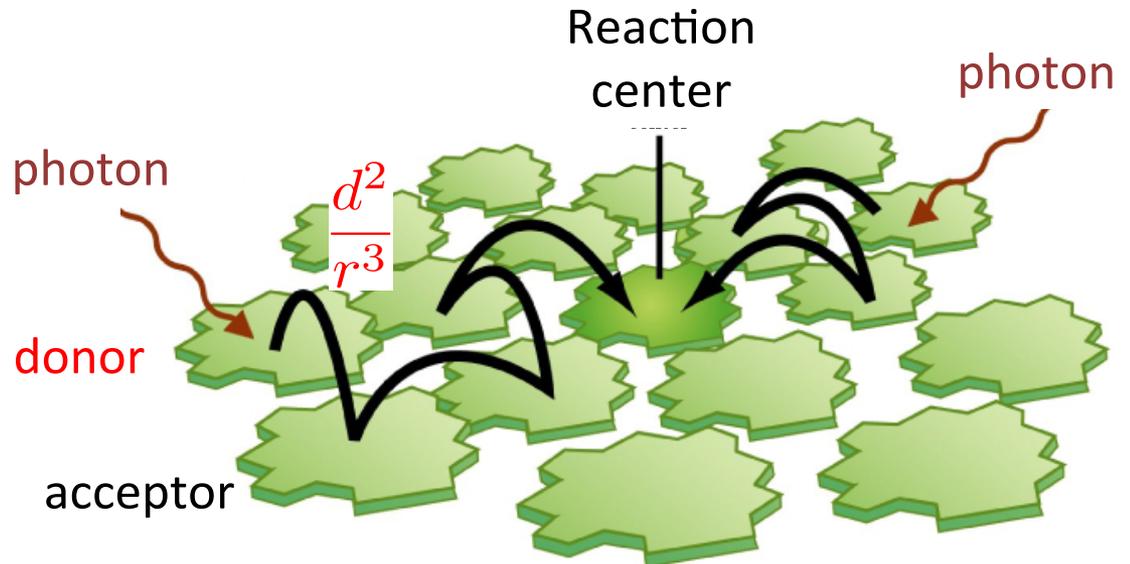
$$k_{tr} = \frac{1}{\tau} \left(\frac{R_0}{r} \right)^6 \propto \left(\frac{d^2}{r^3} \right)^2$$

F. Perrin (1933), Oppenheimer (1941)
Th. Förster (1946)

Clegg, *The History of FRET* (2006)

Photosynthesis:

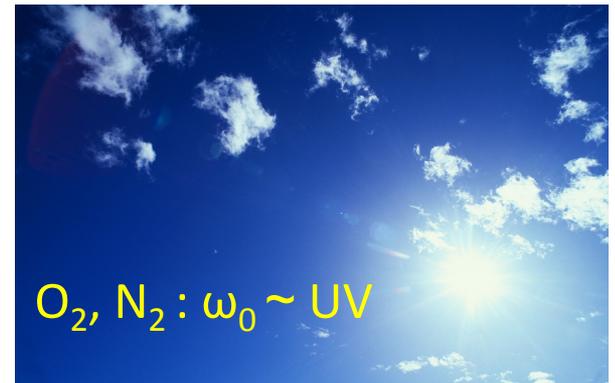
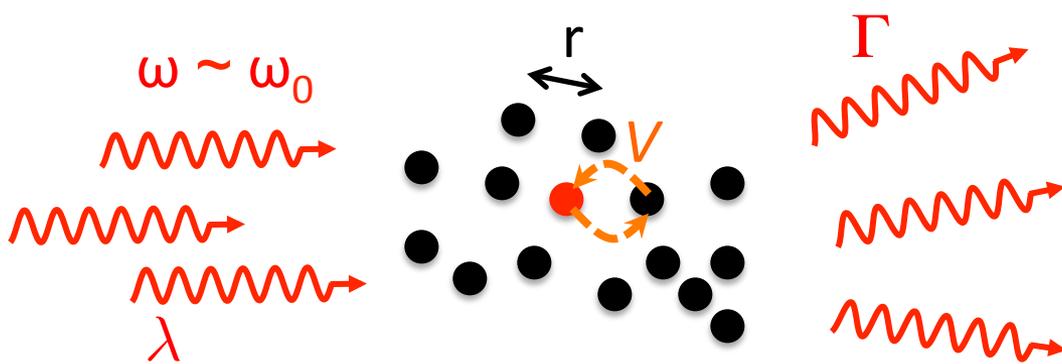
QE > 90%



Resonant interaction and light scattering...

Near-resonance light scattering in dense media

Ensemble of two level-atoms (frequency ω_0 , linewidth Γ)



Non-radiative energy redistribution. Rate: $\frac{V}{\hbar} \Rightarrow$ modifies scattering

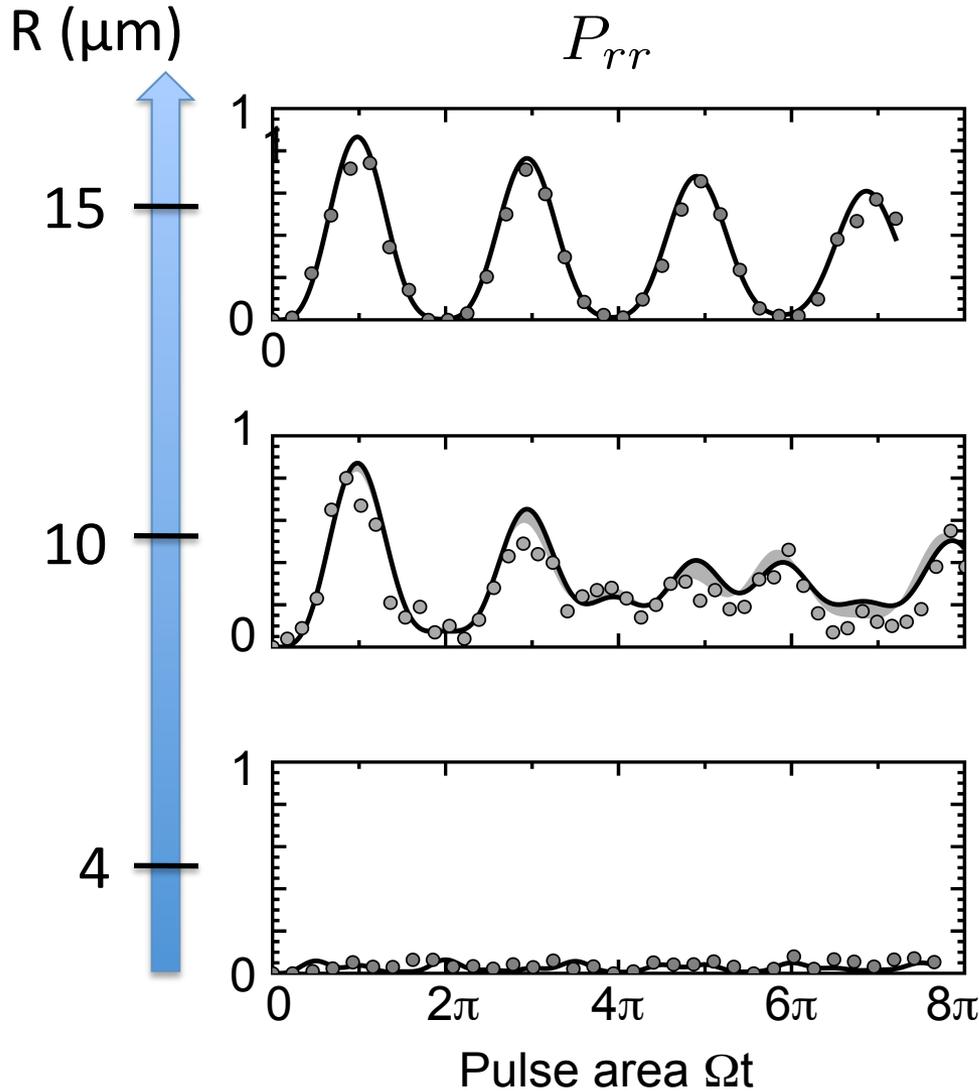
Pellegrino, PRL **113**, 133602 (2014)

Jennewein, PRL **116**, 233601 (2016)

Schilder, PRA **93**, 063835 (2016)

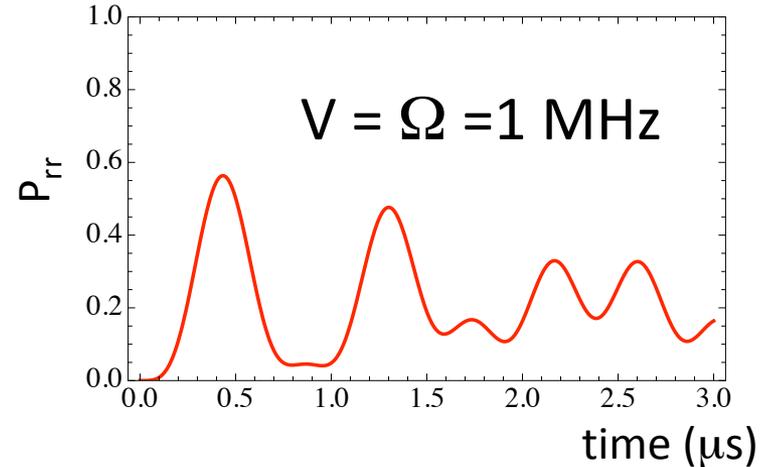
Schilder, PRA **96**, 013825 (2017)

From blockade to many-body physics with 2 atoms ($62d_{3/2}$)



$$\hbar\Omega \gg U_{\text{vdW}}$$

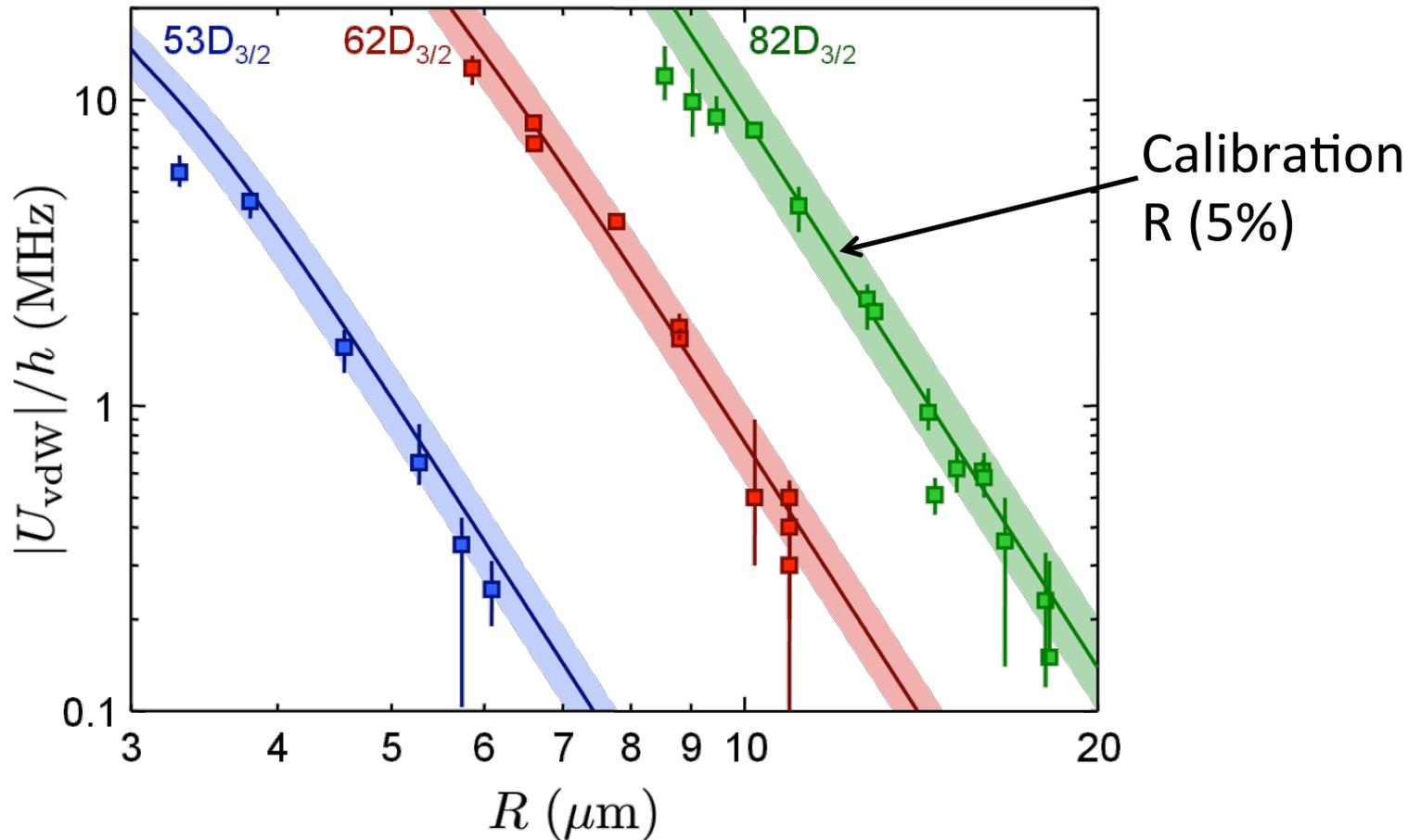
Theory (Schrödinger eq.)



$$\hbar\Omega \ll U_{\text{vdW}}$$

Fit \Rightarrow extract U_{vdW}

Measurement of vdW interaction between 2 atoms

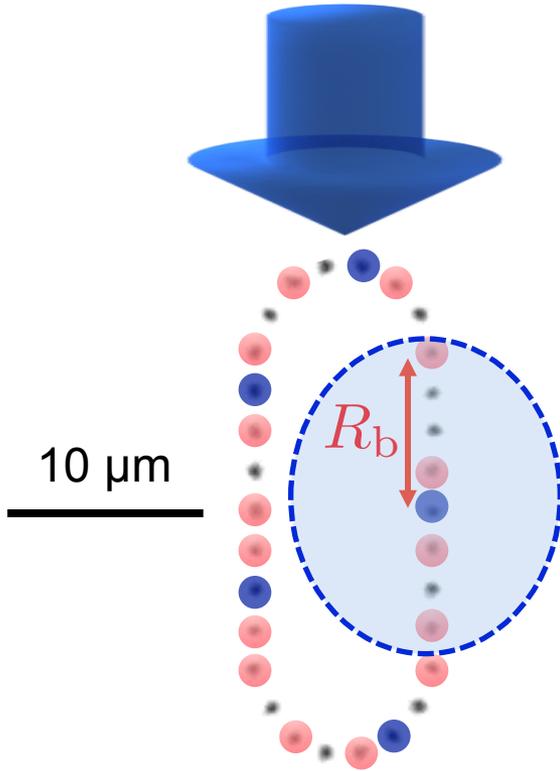


Theory curves: direct diagonalization (dipole-dipole interaction)
No adjustable parameter

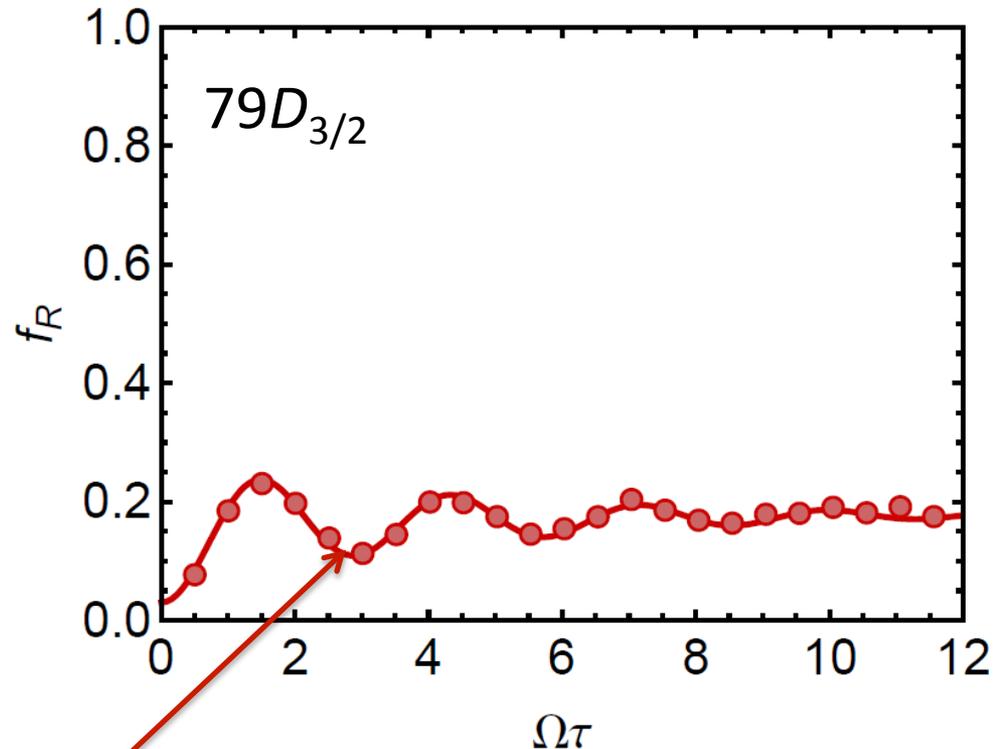
A 1D Ising chain (periodic cond.): mean number of Rydberg excitations

Labuhn *et al.*, Nature **534**, 667 (2016)

Partially loaded 1D ring (30 traps, 20 atoms)



Rydberg fraction: $f_r = \frac{\langle N_r \rangle}{N} \sim \text{magnetization}$

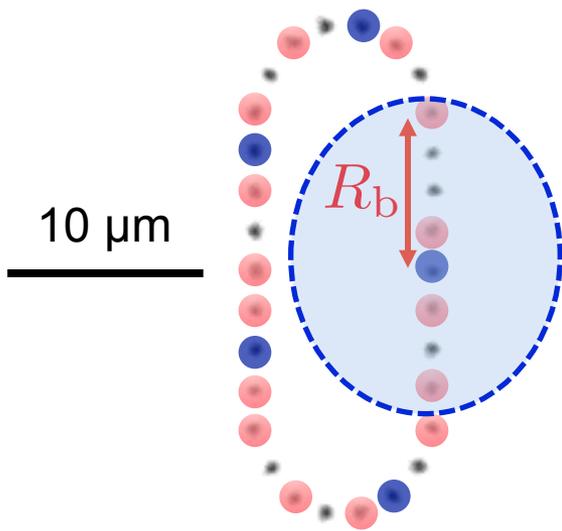


Theory (Schrödinger), no adjustable parameter...!
Includes detection efficiency (T. Macri)

A 1D Ising chain (periodic cond.): pair correlation function

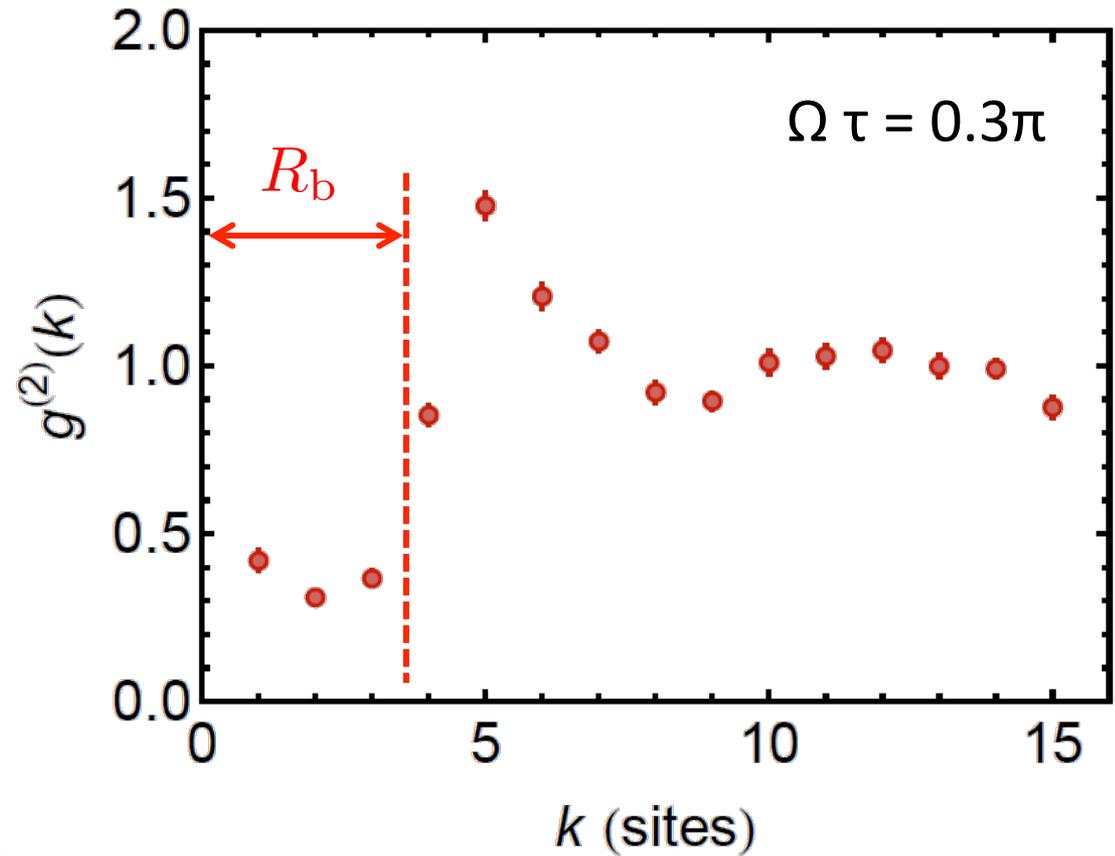
Labuhn *et al.*, Nature **534**, 667 (2016)

Partially loaded 1D ring (30 traps, 20 atoms)



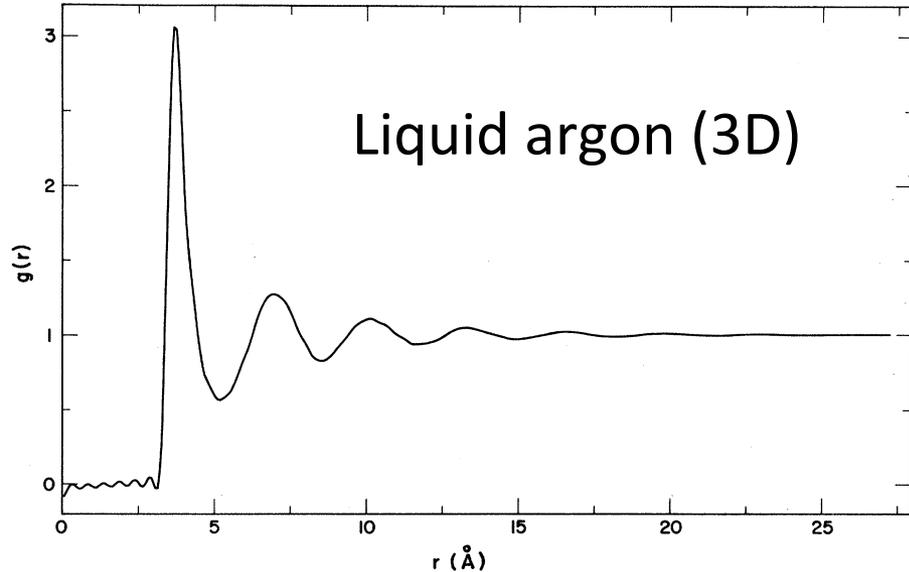
Pair correlation function:

$$g^{(2)}(k) = \frac{1}{N_{\text{tot}}} \sum_i \frac{\langle n_i n_{i+k} \rangle}{\langle n_i \rangle \langle n_{i+k} \rangle}$$



A 1D Ising chain (periodic cond.) = 1D liquid!

Labuhn *et al.*, Nature **534**, 667 (2016)

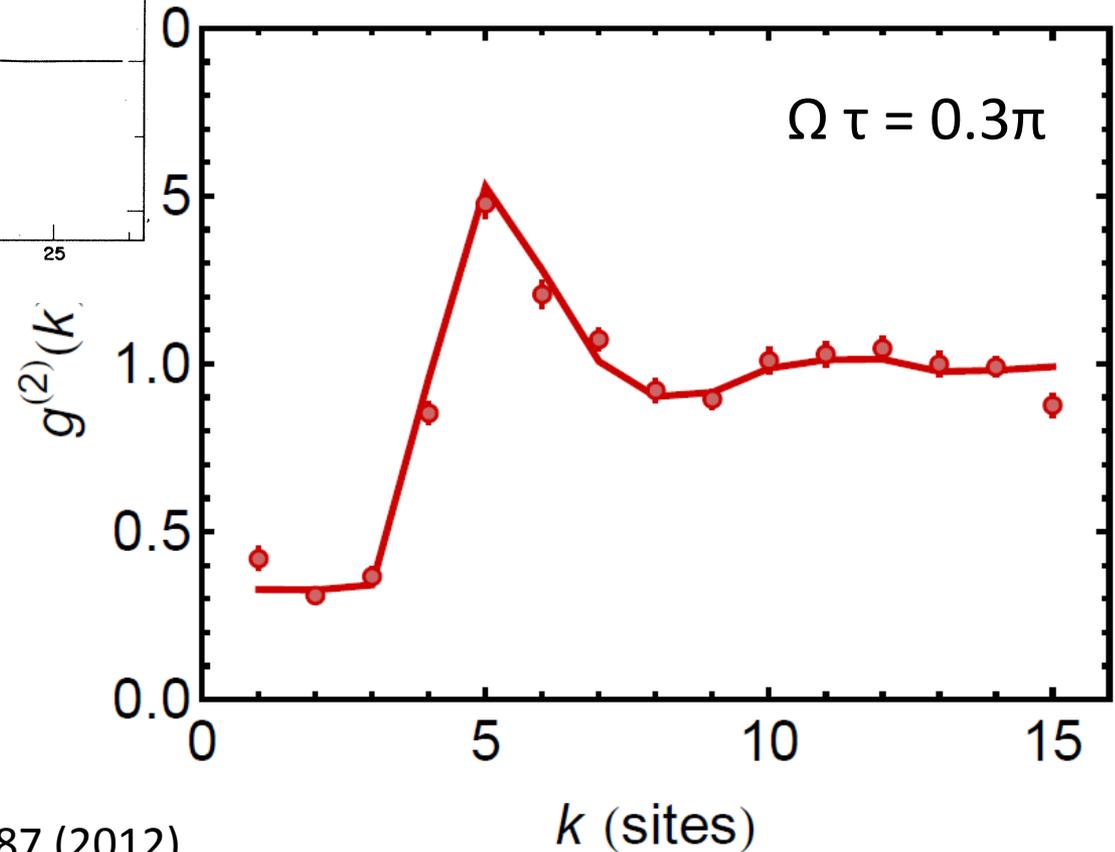


Liquid argon (3D)

PRA **7**, 2130 (1973)

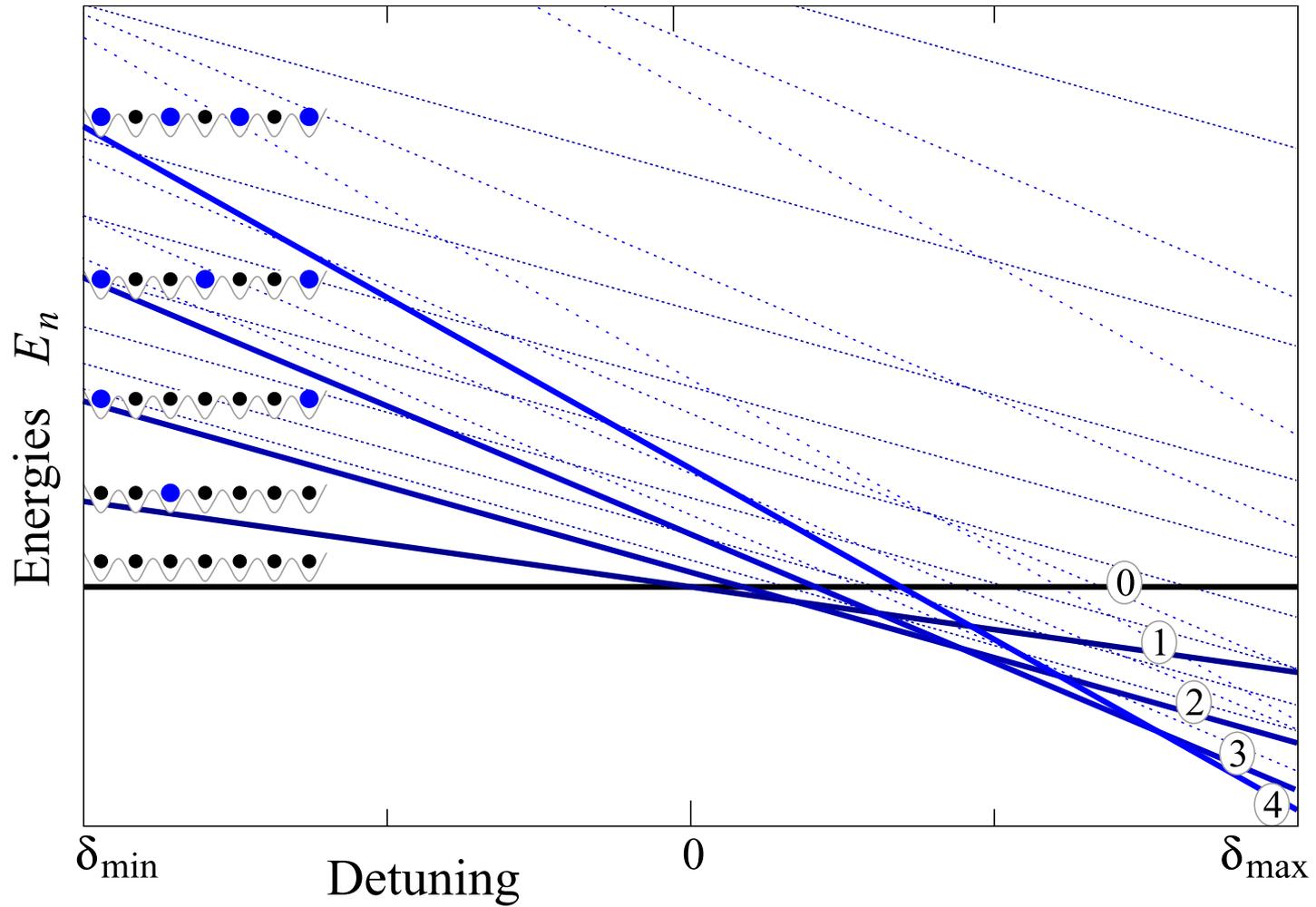
Analogous to 1D liquid:

hard sphere R_b

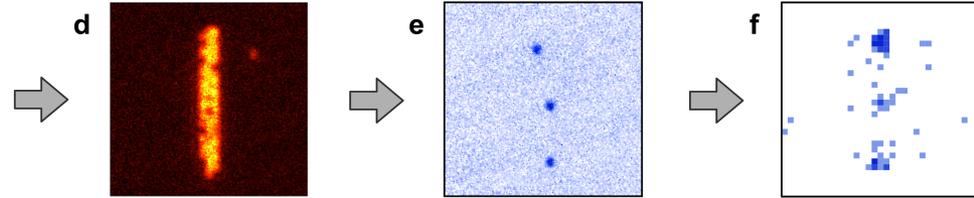
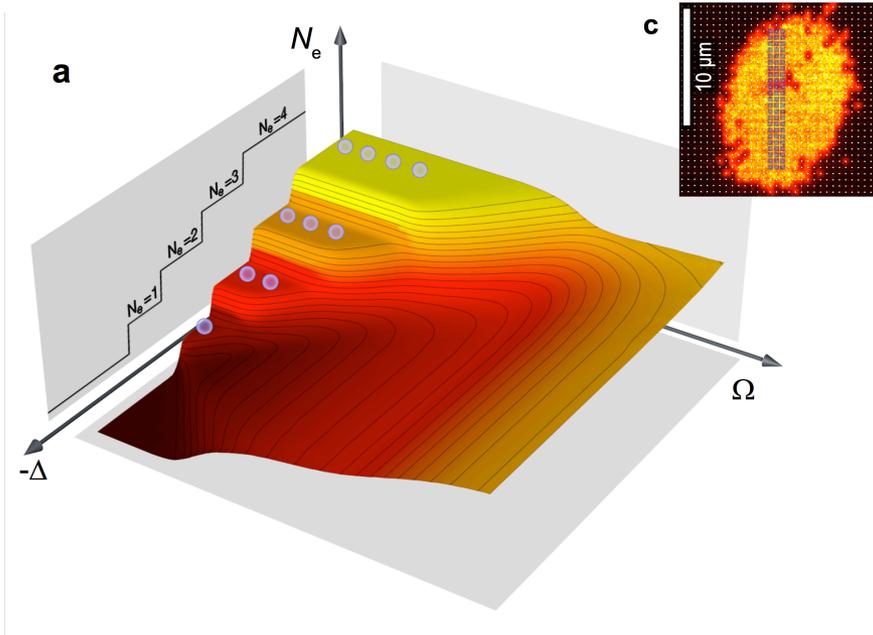


Also in Munich (2D), Nature **491**, 87 (2012)
Ates & Lesanovsky, PRA **86**, 013408 (2012)
Petrosyan, PRA **87** 053414 (2013)

Adiabatic preparation of spatially-ordered 1D Rydberg chains

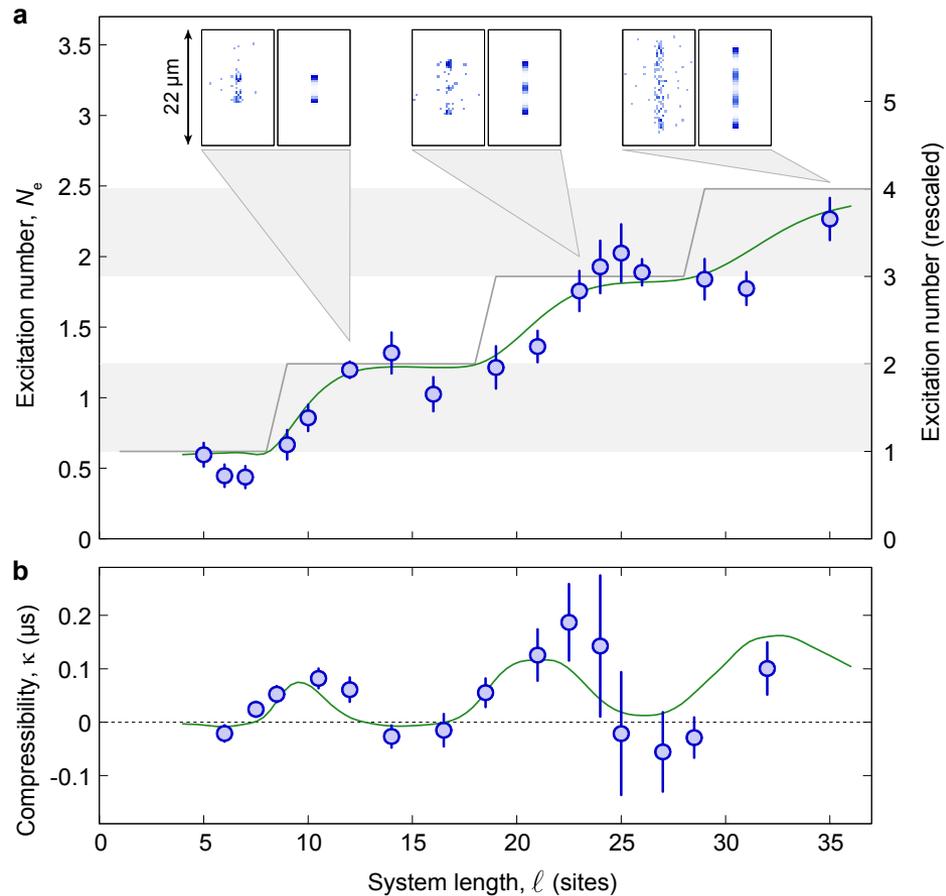


Dynamical crystallization in 1D (MPQ)



$$R_b \gg a$$

Schauss *et al.*, Science (2015)



A 51-atom “quantum simulator” (Harvard-MIT)

$$R_b \sim a$$

H. Bernien, arXiv:1707.04344

